

# Circles

## Directions:

**Assertion (A)** is followed by a statement

of Reason (R). Select the correct option from the following options:

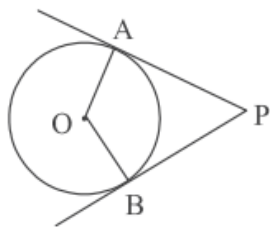
(a) Both, Assertion (A) and Reason (R) are true. Reason (R) explains Assertion (A) completely.

(b) Both, Assertion (A) and Reason (R) are true. Reason (R) does not explain Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

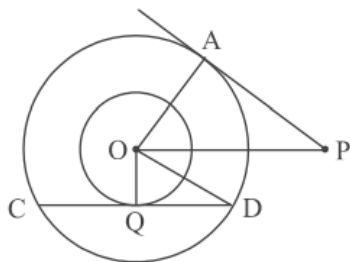
**1. Assertion (A):** If PA and PB are tangents drawn to a circle with centre O from an external point P, then the quadrilateral OAPB is a cyclic quadrilateral.



**Reason (R):** In cyclic quadrilateral opposite angles are equal. (2024)

**Answer.** (c) Assertion (A) is true but Reason (R) is false.

**2. (A)** In two concentric circles, the radii are  $OA = r$  cm and  $OQ = 6$  cm, as shown in the figure. Chord CD of larger circle is a tangent to smaller circle at Q. PA is tangent to larger circle. If  $PA = 16$  cm and  $OP = 20$  cm, find the length CD. (2024)



**Answer.** (A) Since  $PA \perp OA$  therefore  $OA^2 = 20^2 - 16^2 = 144$

$\Rightarrow OA = r = 12$  cm

In  $\triangle OQD$ ,  $QD^2 = 12^2 - 6^2 = 108$



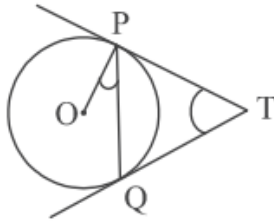
$$\Rightarrow QD = 6\sqrt{3} \text{ cm}$$

Now OQ bisects CD

$$\Rightarrow CD = 2 \times 6\sqrt{3} = 12\sqrt{3} \text{ cm}$$

OR

(B) In given figure, two tangents PT and QT are drawn to a circle with centre O from an external point T. Prove that  $\angle PTQ = 2 \angle OPQ$ . (2024)



**Answer.** (B) Let  $\angle PTQ = \theta$

In  $\triangle TPQ$ ,  $\angle PQT = \angle QPT$  and  $\angle PQT + \angle QPT + \angle PTQ = 180^\circ$

$$\Rightarrow \angle QPT = 90^\circ - \frac{\theta}{2}$$

Now  $OP \perp PT \Rightarrow \angle OPQ + \angle QPT = 90^\circ$

$$\Rightarrow \angle OPQ = \frac{\theta}{2}$$

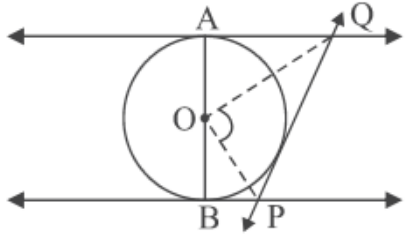
$$\Rightarrow \angle PTQ = 2 \angle OPQ.$$

3. A solid sphere is cut into two hemispheres. The ratio of the surface areas of sphere to that of two hemispheres taken together, is : (2024)

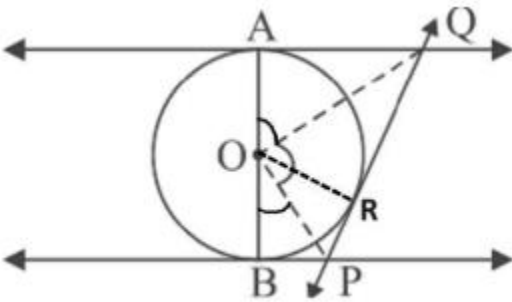
- (a) 1:1
- (b) 1:4
- (c) 2:3
- (d) 3:2

**Answer.** (c) 2 : 3

4. (A) In the given figure, AB is a diameter of the circle with centre O. AQ, BP and PQ are tangents to the circle. Prove that  $\angle POQ = 90^\circ$ . (2024)



Answer.



Join OR.

$$\Delta AOQ \cong \Delta ROQ \Rightarrow \angle AOQ = \angle ROQ \text{ ----- (i)}$$

$$\Delta BOP \cong \Delta ROP \Rightarrow \angle BOP = \angle ROP \text{ ----- (ii)}$$

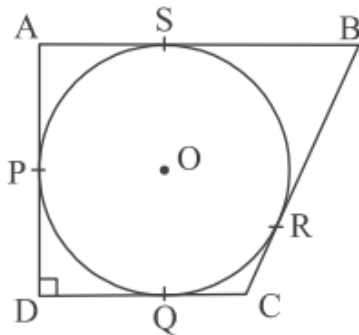
$$\text{Since } \angle AOR + \angle ROB = 180^\circ$$

$$\Rightarrow 2\angle QOR + 2\angle ROP = 180^\circ$$

$$\Rightarrow \angle QOR + \angle ROP = \angle POQ = 90^\circ$$

OR

(B) A circle with centre O and radius 8 cm is inscribed in a quadrilateral ABCD in which P, Q, R, S are the points of contact as shown. If AD is perpendicular to DC,  $BC = 30$  cm and  $BS = 24$  cm, then find the length DC. (2024)



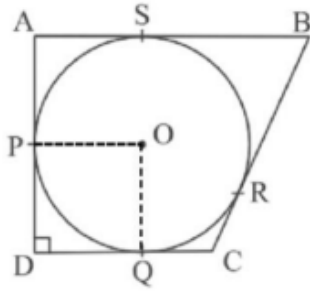
Answer. Join OP and OQ.

$$BR = BS = 24 \text{ cm}$$

$$\therefore CR = 6 \text{ cm}$$

$$\Rightarrow CQ = 6 \text{ cm}$$

Also,  $DQ = OP = 8$  cm  
Hence,  $DC = 8 + 6 = 14$  cm



## 10.1 Introduction

### MCQ

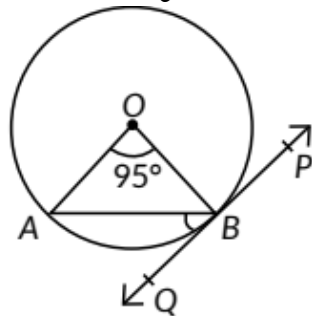
1. A chord of a circle of radius 10 cm subtends a right angle at its centre. The length of the chord (in cm) is

- (a)  $5\sqrt{2}$  (b)  $10\sqrt{2}$   
(c)  $\frac{5}{\sqrt{2}}$  (d)  $10\sqrt{3}$  (AI 2014)

## 10.2 Tangent to a Circle

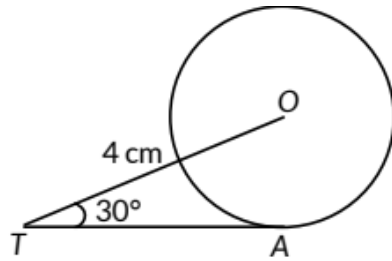
### MCQ

2. In the given figure, PQ is tangent to the circle centred at O. If  $\angle AOB = 95^\circ$ , then the measure of  $\angle ABQ$  will be



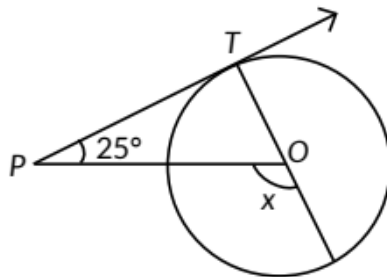
- (a)  $47.5^\circ$  (b)  $42.5^\circ$  (c)  $85^\circ$  (d)  $95^\circ$   
(2023)

3. In the given figure, TA is a tangent to the circle with centre O such that  $OT = 4$  cm,  $\angle OTA = 30^\circ$ , then length of TA is



- (a)  $2\sqrt{3}$  cm                      (b) 2 cm  
 (c)  $2\sqrt{2}$  cm                      (d)  $\sqrt{3}$  cm                      (2023)

4. In the given figure, PT is a tangent at T to the circle with centre O. If  $\angle TPO = 25^\circ$ , then x is equal to



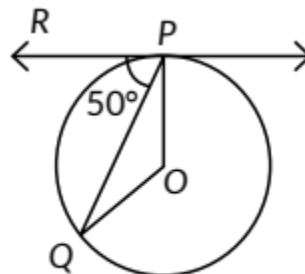
- (a)  $25^\circ$     (b)  $65^\circ$     (c)  $90^\circ$     (d)  $115^\circ$   
 (2023)

5. The length of tangent drawn to a circle of radius 9 cm from a point 41 cm from the centre is

- (a) 40 cm  
 (b) 9 cm  
 (c) 41 cm  
 (d) 50 cm (2023)

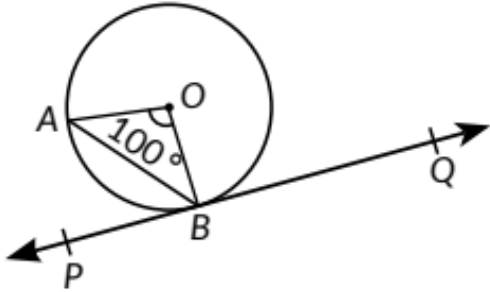
6. In the given figure, O is the centre of the circle and PQ is the chord. If the tangent PR at P makes an angle of  $50^\circ$  with PQ, then the measure of  $\angle POQ$  is

- (a)  $50^\circ$   
 (b)  $40^\circ$   
 (c)  $100^\circ$   
 (d)  $130^\circ$



(2023)

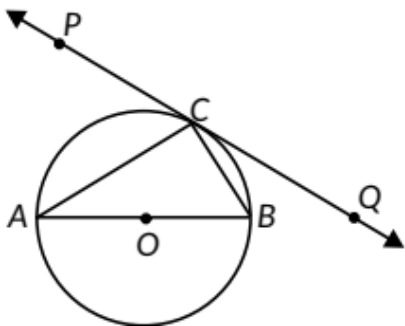
7. In figure, PQ is tangent to the circle with centre at O, at the point B. If  $\angle AOB = 100^\circ$ , then  $\angle ZABP$  is equal to



- (a)  $50^\circ$
- (b)  $40^\circ$
- (c)  $60^\circ$
- (d)  $80^\circ$  (2020)

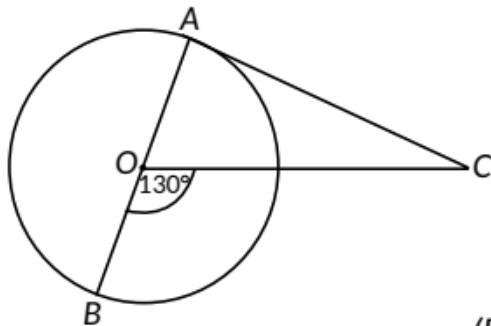
**VSA (1 mark)**

8. In the given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and  $\angle CAB = 30^\circ$ , find  $\angle PCA$ .



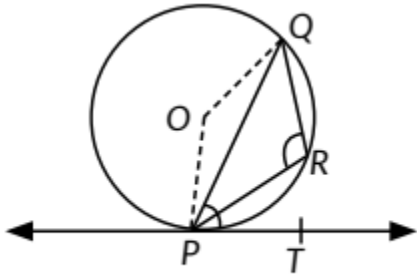
(AI 2016)

9. In the given figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A.  $\angle BOC = 130^\circ$ , then find  $\angle ACO$ .



(Foreign 2016)

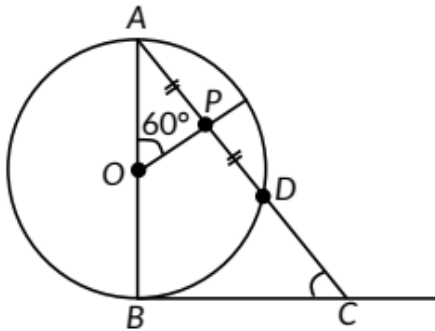
10. In the given figure, PQ is a chord of a circle with centre O and PT is a tangent. If  $\angle QPT = 60^\circ$ , find  $\angle PRQ$ .



(AI 2015)

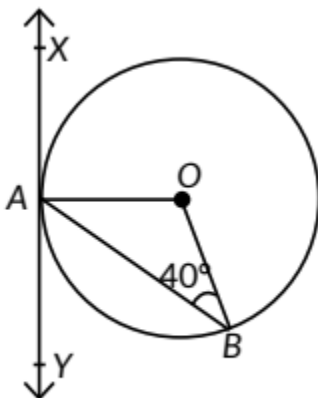
SAI (2 marks)

11. In Fig. AB is diameter of a circle centered at O. BC is tangent to the circle at B. If OP bisects the chord AD and  $\angle AOP = 60^\circ$ , then find  $m\angle C$ .



(Term II, 2021-22)

12. In Fig. XAY is a tangent to the circle centred at O. If  $\angle ABO = 40^\circ$ , then find  $m\angle BAY$  and  $m\angle AOB$ .



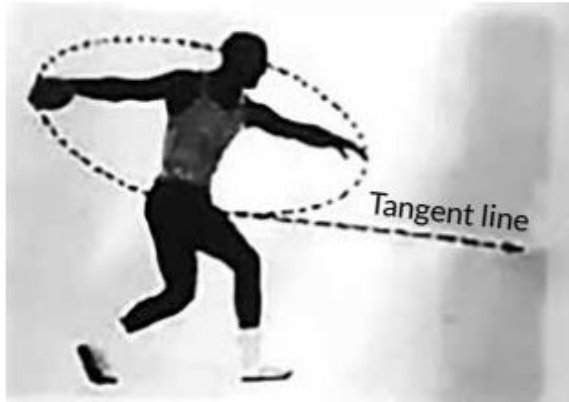
(Term II, 2021-22)



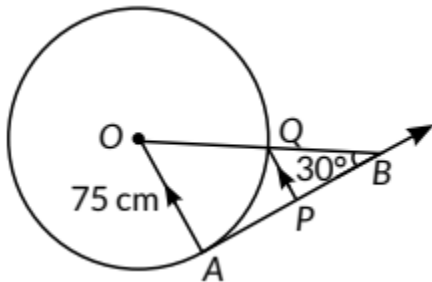
**LA (4/5/6 marks)**

(In this section, there are 3 case study/passage based questions. Each question is of 4 marks.)

14. Case Study: The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.



In the given figure, AB is one such tangent to a circle of radius 75 cm. Point O is centre of the circle and  $\angle ABO = 30^\circ$ . PQ is parallel to OA.



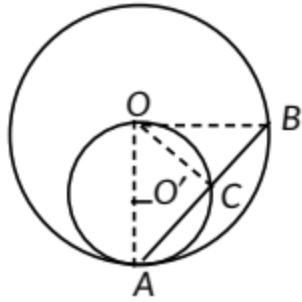
Based on above information:

- (a) Find the length of AB.
- (b) Find the length of OB.
- (c) Find the length of AP.

**OR**

Find the length of PQ. (2023)

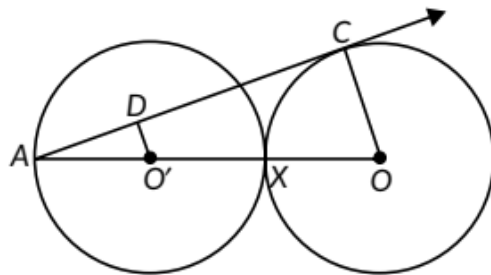
15. In Figure, two circles with centres at O and O' of radii  $2r$  and  $r$  respectively, touch each other internally at A. A chord AB of the bigger circle meets the smaller circle at C. Show that C bisects AB.



(Term II, 2021-22)

16. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. (Delhi 2016, 2015, 2014, AI 2016, 2015, 2014, Foreign 2016, 2015, 2014)

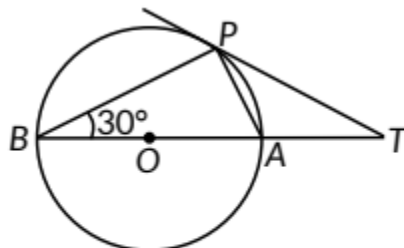
17. In the given figure, two equal circles, with centres  $O$  and  $O'$  touch each other at  $X$ .  $OO'$  produced meets the circle with centre  $O'$  at  $A$ .  $AC$  is tangent to the circle with centre  $O$ , at the point  $C$ .  $O'D$  is perpendicular to  $AC$ . Find the value of  $\frac{DO'}{CO}$ .



(AI 2016)

18. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc. (AI 2015)

19. In the given figure,  $O$  is the centre of the circle and  $TP$  is the tangent to the circle from an external point  $T$ .



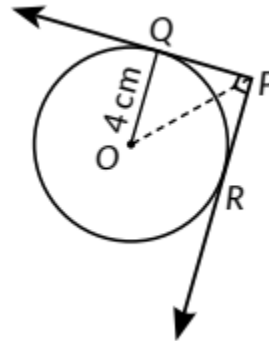
If  $\angle PBT = 30^\circ$ , prove that  $BA : AT = 2 : 1$ .

(Foreign 2015)

### 10.3 Number of Tangents from a Point on a Circle

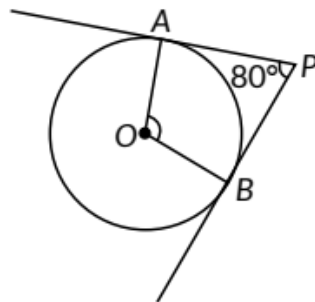
#### MCQ

20. In figure, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If  $\angle QPR = 90^\circ$ , then length of PQ is



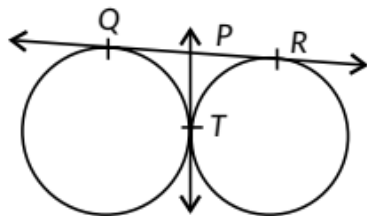
- (a) 3 cm
- (b) 4 cm
- (c) 2 cm
- (d)  $2\sqrt{2}$  cm (2020)

21. In Figure, if tangents PA and PB from an external point P to a circle with centre O, are inclined to each other at an angle of  $80^\circ$ , then  $\angle AOB$  is equal to



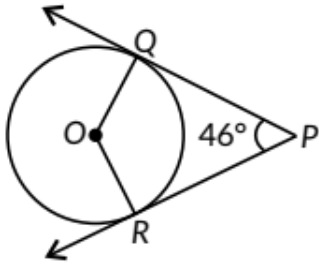
- (a)  $100^\circ$
- (b)  $60^\circ$
- (c)  $80^\circ$
- (d)  $50^\circ$   
(2020)

22. In the given figure, QR is a common tangent to the given circles, touching externally at the point T. The tangent at T meets QR at P. If  $PT = 3.8$  cm, then the length of QR (in cm) is



- (a) 3.8
- (b) 7.6
- (c) 5.7
- (d) 1.9  
(Delhi 2014)

23. In the given figure, PQ and PR are two tangents to a circle with centre O. If  $\angle QPR = 46^\circ$ , then  $\angle QOR$  equals

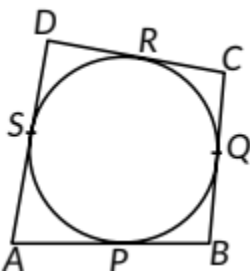


- (a)  $67^\circ$
- (b)  $134^\circ$
- (c)  $44^\circ$
- (d)  $46^\circ$  (Delhi 2014)

24. Two circles touch each other externally at P. AB is a common tangent to the circles touching them at A and B. The value of  $\angle APB$  is

- (a)  $30^\circ$
- (b)  $45^\circ$
- (c)  $60^\circ$
- (d)  $90^\circ$  (AI 2014)

25. In the given figure, a quadrilateral ABCD is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. If  $AB = x$  cm,  $BC = 7$  cm,  $CR = 3$  cm and  $AS = 5$  cm, find  $x$ .



- (a) 10
- (b) 9
- (c) 8
- (d) 7 (Foreign 2014)

26. Two concentric circles are of radii 5 cm and 3 cm. Length of the chord of the larger circle, (in cm), which touches the smaller circle is

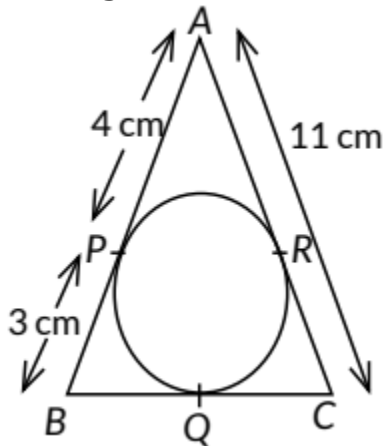
- (a) 4

- (b) 5
- (c) 8
- (d) 10 (Foreign 2014)

**VSA (1 mark)**

27. If tangents PA and PB from an external point P to a circle with centre O are inclined to each other at an angle of  $70^\circ$ , then find  $\angle POA$ . (2021)

28. In figure, ABC is circumscribing a circle, the length of BC is \_\_\_\_\_ cm.

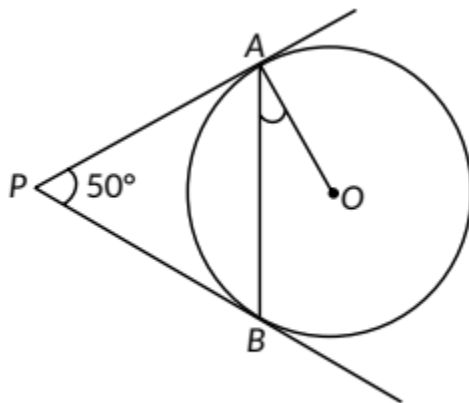


(2020)

29. If the angle between two tangents drawn from an external point P to a circle of radius  $a$  and centre O, is  $60^\circ$ , then find the length of OP. (AI 2017)

30. From an external point P, tangents PA and PB are drawn to a circle with centre O. If  $\angle PAB = 50^\circ$ , then find  $\angle AOB$ . (Delhi 2016)

31. In the given figure, PA and PB are tangents to the circle with centre O such that  $\angle APB = 50^\circ$ . Write the measure of  $\angle OAB$ .

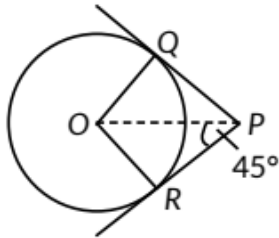


(Delhi 2015)

32. Two concentric circles of radii  $a$  and  $b$  ( $a > b$ ) are given. Find the length of the chord of the larger circle which touches the smaller circle. (Foreign 2015)

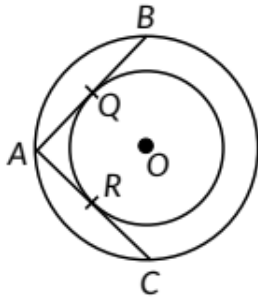
**SAI (2 marks)**

33. In Figure,  $PQ$  and  $PR$  are tangents to the circle centred at  $O$ . If  $\angle ZOPR = 45^\circ$ , then prove that  $ORPQ$  is a square.



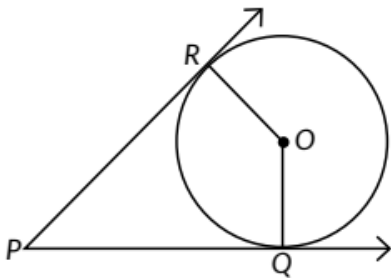
(Term II, 2021-22)

34. In Fig., there are two concentric circles with centre  $O$ . If  $ARC$  and  $AQB$  are tangents to the smaller circle from the point  $A$  lying on the larger circle, find the length of  $AC$ , if  $AQ = 5$  cm.



(Term II, 2021-22,

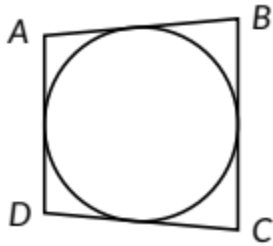
35. In Figure,  $O$  is the centre of the circle.  $PQ$  and  $PR$  are tangent segments. Show that the quadrilateral  $PQOR$  is cyclic.



(Term II, 2021-22)

36. In figure, a quadrilateral  $ABCD$  is drawn to circumscribe a circle. Prove that

$$AB + CD = BC + AD$$



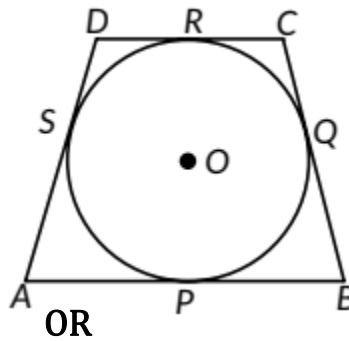
(2020)

OR

A circle touches all the four sides of a quadrilateral ABCD. Prove that  $AB + CD = BC + DA$  (AI 2017)

OR

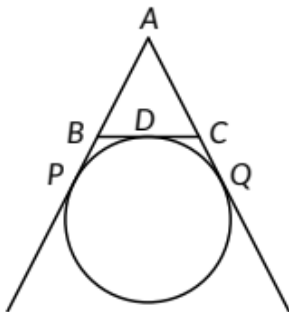
In the given figure, a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that  $AB + CD = BC + DA$ . (AI 2016)



OR

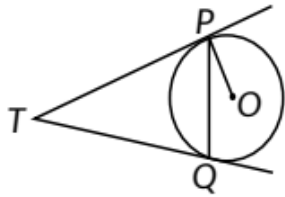
A quadrilateral is drawn to circumscribe a circle. Prove that the sums of opposite sides are equal. (Foreign 2014)

37. In figure, find the perimeter of  $\triangle ABC$ , if  $AP = 12$  cm.



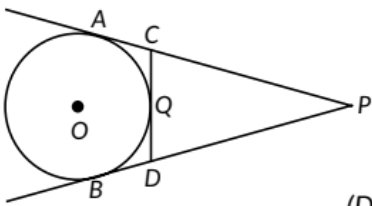
(2020)

38. In figure, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that  $\angle PTQ = 2\angle OPQ$ .



(2020)

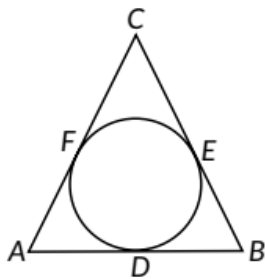
39. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If  $PA = 12$  cm,  $QC = QD = 3$  cm then find  $PC + PD$ .



(Delhi 2017)

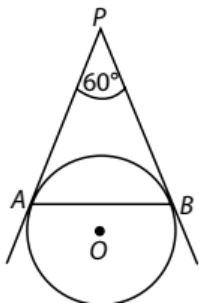
40. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord. (AI 2017)

41. In the given figure, a circle is inscribed in a  $\triangle ABC$ , such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF.



(Delhi 2016)

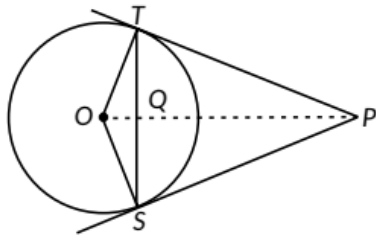
42. In the given figure, AP and BP are tangents to a circle with centre O, such that  $AP = 5$  cm and  $\angle APB = 60^\circ$ . Find the length of chord AB.



(Delhi 2016)

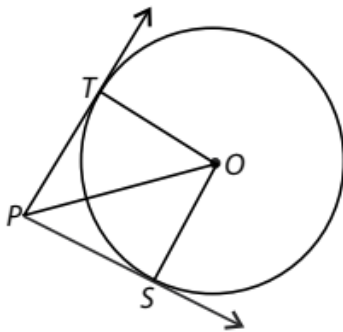


43. In the given figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If  $OP=2r$ , show that  $\angle ZOTS = \angle ZOST = 30^\circ$ .



(AI 2016)

44. In the given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that  $\angle SPT = 120^\circ$ . Prove that  $OP = 2PS$ .

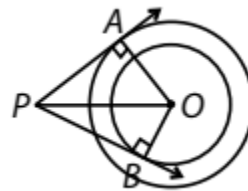


(Foreign 2016,

OR

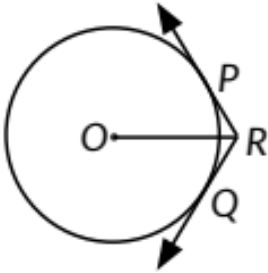
If from an external point P of a circle with centre O, two tangents PQ and PR are drawn such that  $\angle QPR = 120^\circ$ , prove that  $2PQ = PO$ . (Delhi 2014)

45. In the given figure, there are two concentric circles of radii 6 cm and 4 cm with centre O. If AP is a tangent to the larger circle and BP to the smaller circle and length of AP is 8 cm, find the length of BP. (Foreign 2016)

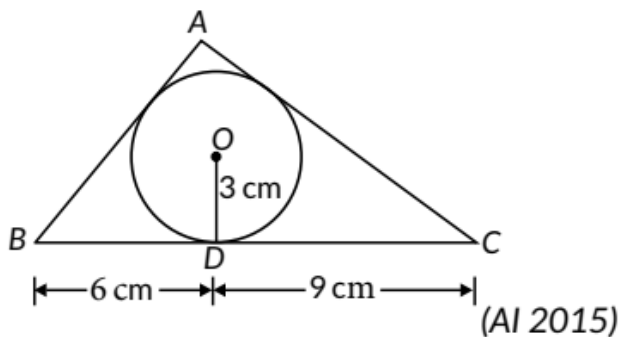


46. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ. (Delhi 2015)

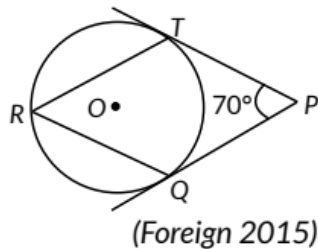
47. In the given figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If  $\angle PRQ = 120^\circ$ , then prove that  $OR = PR + RQ$ . (AI 2015)



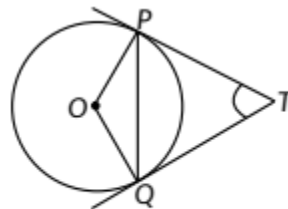
48. In the given figure, a  $\Delta ABC$  is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of  $\Delta ABC$  is  $54 \text{ cm}^2$ , then find the lengths of sides AB and AC



49. In the given figure O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If  $\angle ZTPQ = 70^\circ$ , find  $\angle TRQ$ .



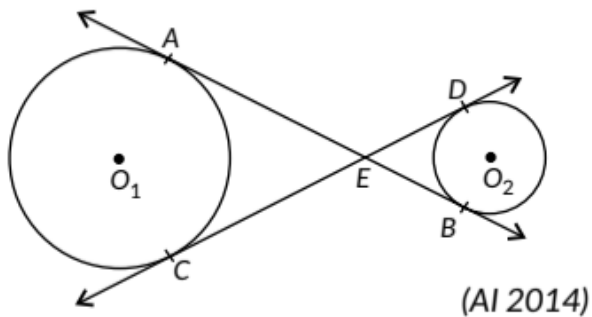
50. In the given figure PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the lengths of TP and TQ.



51. Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre. (Delhi 2014)

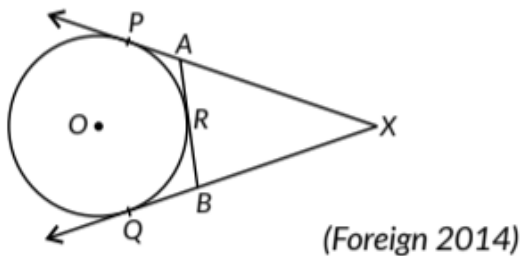
52. In the given figure, common tangents AB and CD to the two circles with centres  $O_1$  and  $O_2$  intersect at E. Prove that  $AB = CD$ .





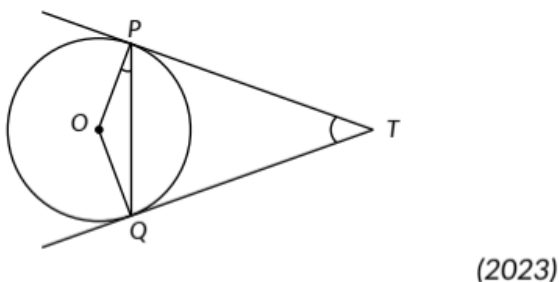
53. The incircle of an isosceles triangle  $ABC$ , in which  $AB = AC$ , touches the sides  $BC$ ,  $CA$  and  $AB$  at  $D$ ,  $E$  and  $F$  respectively. Prove that  $BD = DC$ . (AI 2014)

54. In the given figure,  $XP$  and  $XQ$  are two tangents to the circle with centre  $O$ , drawn from an external point  $X$ .  $ARB$  is another tangent, touching the circle at  $R$ . Prove that  $XA + AR = XB + BR$ .



**SA II (3 marks)**

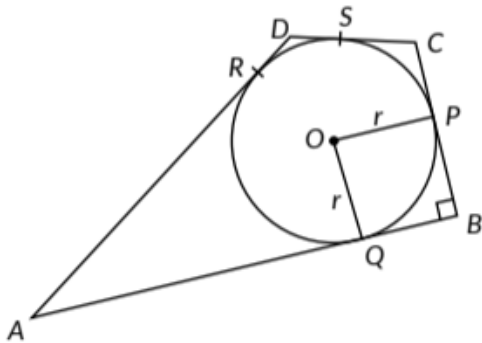
55. Two tangents  $TP$  and  $TQ$  are drawn to a circle with centre  $O$  from an external point  $T$ . Prove that  $\angle PTQ = 2\angle OPQ$ .



**OR**

Two tangents  $TP$  and  $TQ$  are drawn to a circle with centre  $O$  from an external point  $T$ . Prove that  $\angle PTQ = 2\angle OPQ$  (Delhi 2017)

56. In the given figure, a circle is inscribed in a quadrilateral ABCD in which  $\angle B = 90^\circ$ . If  $AD = 17$  cm,  $AB = 20$  cm and  $DS = 3$  cm, then find the radius of the circle.



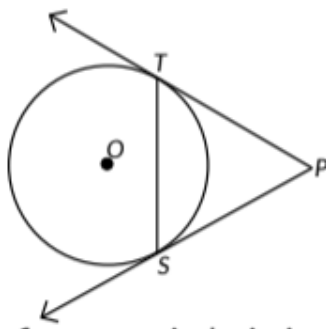
(2023)

57. From an external point, two tangents are drawn to a circle. Prove that the line joining the external point to the centre of the circle bisects the angle between the two tangents. (2023)

58. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. (2023)

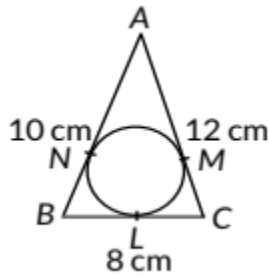
59. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre. (2023)

60. In the given figure, PT and PS are tangents to a circle with centre O, from a point P, such that  $PT = 4$  cm and  $\angle TPS = 60^\circ$ . Find the length of the chord TS. Also, find the radius of the circle.



(2021)

61. In the given figure, a circle is inscribed in a  $\triangle ABC$  having sides  $BC = 8$  cm,  $AB = 10$  cm and  $AC = 12$  cm. Find the lengths  $BL$ ,  $CM$  and  $AN$ .



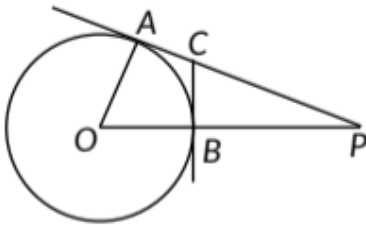
(Delhi 2019)

62. Prove that tangents drawn at the ends of a diameter of a circle are parallel.

63. (AI 2019, Delhi 2017, Foreign 2014)

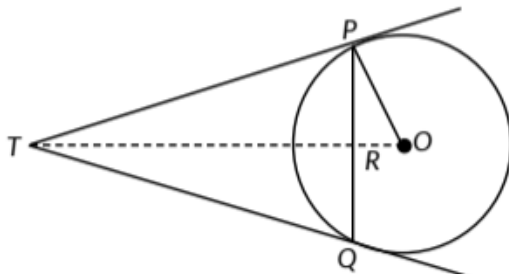
LA(4/5/6 marks)

63. In Figure  $O$  is centre of a circle of radius 5 cm.  $PA$  and  $BC$  are tangents to the circle at  $A$  and  $B$  respectively. If  $OP$  13 cm, then find the length of tangents  $PA$  and  $BC$ .



(Term II, 2021-22)

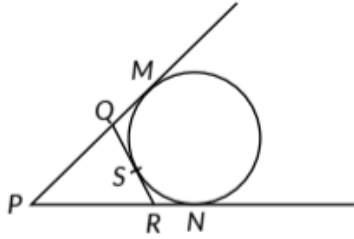
64. In fig.  $PQ$  is a chord of length 8 cm of a circle of radius 5 cm. The tangents at  $P$  and  $Q$  meet at a point  $T$ . Find the length of  $TP$ .



(Term II, 2021-22)

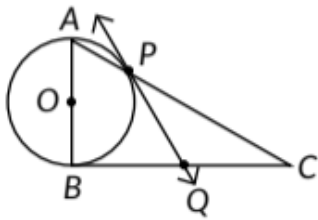
65. Prove that a parallelogram circumscribing a circle is a rhombus. (Term II, 2021-22, Delhi 2014)

66. In fig, if a circle touches the side  $QR$  of  $\triangle PQR$  at  $S$  and extended sides  $PQ$  and  $PR$  at  $M$  and  $N$ , respectively, then



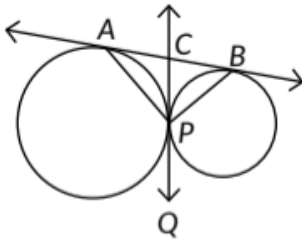
Prove that  $PM = \frac{1}{2}(PQ + QR + PR)$  (Term II, 2021-22,

67. In figure, a triangle ABC with  $\angle B = 90^\circ$  is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point P. Prove that the tangent drawn at point P bisects BC.



(Term II, 2021-22)

68. In figure, two circles touch externally at P. A common tangent touches them at A and B and another common tangent is at P, which meets the common tangent AB at C. Prove that  $\angle APB = 90^\circ$ .

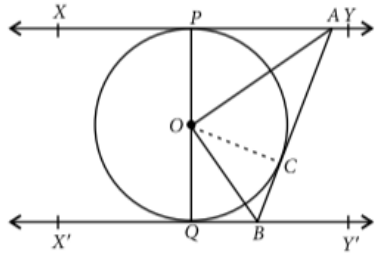


(Term II, 2021-22)

69. Prove that the length of tangents drawn from an external point to a circle are equal. (2018, Delhi 2017, 2016, 2015, 2014, AI 2017, 2016, 2015, Foreign 2016, 2015, 2014)

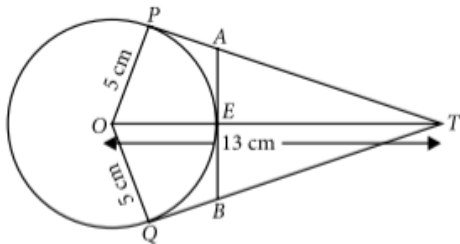
70. In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, is intersecting XY at

A and X'Y' at B. Prove that  $\angle AOB = 90^\circ$ .



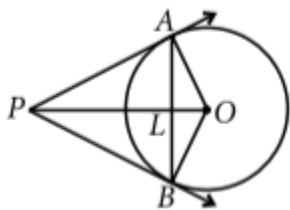
(AI 2017)

71. In the given figure, O is the centre of a circle of radius 5 cm. T is a point such that  $OT = 13$  cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



(Delhi 2016)

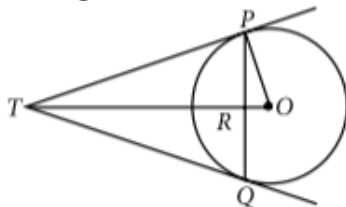
72. In the given figure, AB is a chord of a circle, with centre O, such that  $AB = 16$  cm and radius of circle is 10 cm. Tangents at A and B intersect each other at P. Find the length of PA.



(Foreign 2016)

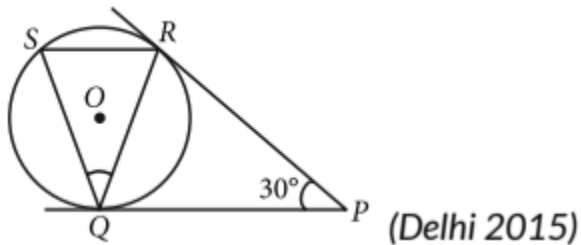
OR

In the given figure, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T. Find the length of TP.



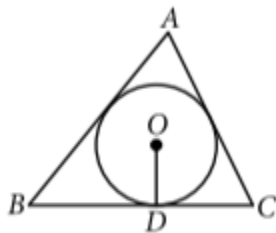
(AI 2014)

73. In the given figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that  $\angle RPQ = 30^\circ$ . A chord RS is drawn parallel to the tangent PQ. Find  $\angle RQS$ .



74. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. (AI 2014)

75. In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 4 cm, such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.



(AI 2014) Ap

## CBSE Sample Questions

### 10.2 Tangent to a Circle

VSA (1 mark)

1. PQ is a tangent to a circle with centre O at point P. If  $\triangle OPQ$  is an isosceles triangle, then find  $\angle OQP$ . (2020-21)

### 10.3 Number of Tangents from a Point on a Circle

MCQ

2. If two tangents inclined at an angle of  $60^\circ$  are drawn to a circle of radius 3 cm, then the length of each tangent is equal to



(a)  $\frac{3\sqrt{3}}{2}$  cm

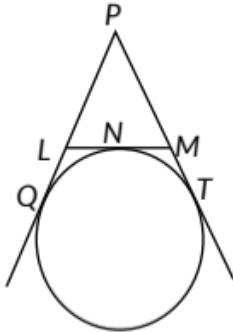
(b) 3 cm

(c) 6 cm

(d)  $3\sqrt{3}$  cm (2022-23)

VSA (1 mark)

3. If  $PQ = 28$  cm, then find the perimeter of  $\triangle PLM$ .



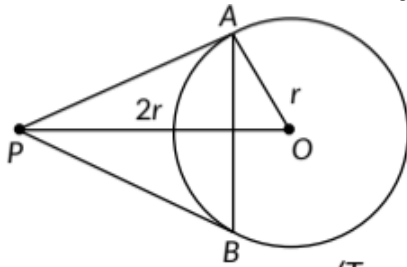
(2020-21)

4. If two tangents are inclined at  $60^\circ$  are drawn to a circle of radius 3 cm, then find length of each tangent. (2020-21)

SAI (2 marks)

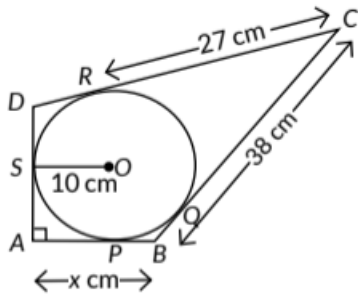
5. In the given figure, O is the centre of circle. Find  $\angle AQB$ , given that PA and PB are tangents to the circle and  $\angle APB = 75^\circ$ .

6. From a point P, two tangents PA and PB are drawn to a circle  $C(O, r)$ . If  $OP = 2r$ , then find  $\angle APB$ . What type of triangle is APB?



(Term II, 2021-22)

7. In the figure, quadrilateral ABCD is circumscribing a circle with centre O and AD AB. If radius of incircle is 10 cm, then the value of x is

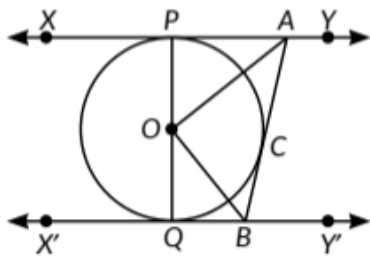


(2020-21)

SA II (3 marks)

8. Prove that a parallelogram circumscribing a circle is a rhombus. (2022-23)

9. In the figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B, what is the measure of  $\angle AOB$ .

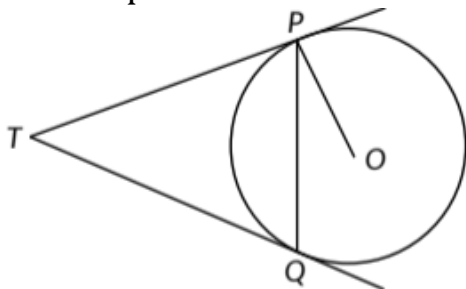


(2022-23)

LA (4/5/6 marks)

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact to the centre. (Term II, 2021-22)

11. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that  $\angle PTQ = 2\angle OPQ$ .



(Term II, 2021-22)

## SOLUTIONS

### Previous Years' CBSE Board Questions

1. (b): Let AB is a chord of circle which subtends right angle at its centre.

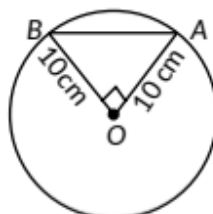
∴ In  $\triangle OAB$ , by Pythagoras theorem, we have

$$(AB)^2 = (OA)^2 + (OB)^2$$

$$\Rightarrow (AB)^2 = (10)^2 + (10)^2$$

$$\Rightarrow (AB)^2 = 200$$

$$\Rightarrow AB = 10\sqrt{2} \text{ cm}$$



2.

(a): We have  $\angle AOB = 95^\circ$

In  $\triangle AOB$ ,  $\angle OAB = \angle OBA$  (since  $OA = OB$ ) ... (i)

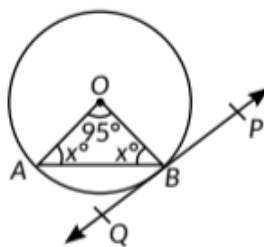
Now,  $\angle OAB + 95^\circ + \angle OBA = 180^\circ$

(Angle sum property of a triangle)

$$\Rightarrow \angle OAB = \frac{85^\circ}{2} = 42.5^\circ$$

∴  $\angle OAB = \angle OBA = 42.5^\circ$  (From (i))

Now,  $OB$  is perpendicular to the tangent line  $PQ$ .



$$= \angle QBO = 90^\circ$$

$$= \angle ABO + \angle ABQ = 90^\circ$$

$$= 42.5^\circ + \angle ABQ = 90^\circ$$

$$= \angle ABQ = 47.5^\circ$$

3. (a): Draw  $OAI TA$ .

In  $\triangle OTA$ ,  $\angle OAT = 90^\circ$  (∵ Tangent to a circle is perpendicular to the radius passing through the point of contact) and  $\angle OTA = 30^\circ$

$$\therefore \frac{TA}{OT} = \cos 30^\circ \Rightarrow TA = 4 \cos 30^\circ = 4 \times \frac{\sqrt{3}}{2}$$

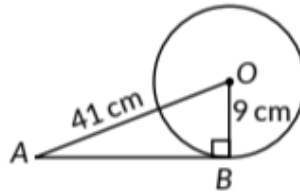
$$TA = 2\sqrt{3} \text{ cm}$$

4. (d): Since,  $\angle TPO = 25^\circ$  and  $\angle OTP = 90^\circ$   
 ( $\because$  Radius is perpendicular to the tangent T)

$$x = \angle OTP + \angle TPO \\ = 90^\circ + 25^\circ = 115^\circ$$

5. (a):  $OB \perp AB$  [As tangent to a circle is perpendicular to the radius through the point of the contact] In  $\triangle OAB$ ,

$$\begin{aligned} OA^2 &= OB^2 + AB^2 && \text{[By Pythagoras theorem]} \\ \Rightarrow 41^2 &= 9^2 + AB^2 \\ \Rightarrow AB^2 &= 41^2 - 9^2 \\ &= (41 - 9)(41 + 9) \\ &= (32)(50) \\ &= 1600 \\ \Rightarrow AB &= \sqrt{1600} = 40 \text{ cm} \end{aligned}$$



6. (c): PR is tangent which touches circle at point P.

So,  $\angle OPR = 90^\circ$

$$\angle OPQ = 90^\circ - \angle RPQ = 90^\circ - 50^\circ = 40^\circ$$

In,  $\triangle POQ$ ,

$OP = OQ$  (Radii of circle)

So,  $\angle OQP = \angle OPQ = 40^\circ$

$$\therefore \angle POQ = 180^\circ - 40^\circ - 40^\circ = 100^\circ$$

7.

(a): In  $\triangle OAB$ ,  $OA = OB$

( $\because$  Radius of a circle are equal)

$$\therefore \angle OAB = \angle OBA \quad \dots(i)$$

( $\because$  Angles opposite to equal sides are equal)

Now, by angle sum property of a triangle

$$\angle OAB + \angle ABO + \angle AOB = 180^\circ$$

$$\Rightarrow \angle ABO + \angle ABO + 100^\circ = 180^\circ \quad \text{(Using (i))}$$

$$\Rightarrow 2\angle ABO = 180^\circ - 100^\circ = 80^\circ$$

$$\Rightarrow \angle ABO = \frac{80^\circ}{2} = 40^\circ$$

Here,  $\angle OBP = 90^\circ$

(Radius is perpendicular to tangent at point of contact)

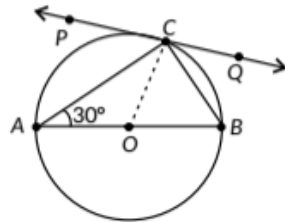
$$\Rightarrow \angle OBA + \angle ABP = 90^\circ$$

$$\Rightarrow 40^\circ + \angle ABP = 90^\circ$$

$$\Rightarrow \angle ABP = 90^\circ - 40^\circ = 50^\circ$$

8. Construction: Join OC

Now, in  $\triangle AOC$ ,  $AO = OC$  (radii of same circle)



$$\Rightarrow \angle OAC = \angle OCA = 30^\circ$$

[Angles opposite to equal sides are equal]

$$\text{Also, } \angle OCP = 90^\circ$$

[Tangent to a circle is  $\perp$  to radius at point of contact]

$$\therefore \angle PCA + \angle OCA = 90^\circ$$

$$\Rightarrow \angle PCA + 30^\circ = 90^\circ \Rightarrow \angle PCA = 60^\circ$$

9.

$$\text{Given, } \angle BOC = 130^\circ$$

Since, AC is a tangent to the circle at A

$$\therefore \angle OAC = 90^\circ \text{ [}\because \text{Radius is perpendicular to the tangent at point of contact]}$$

Now,  $\angle AOC + \angle BOC = 180^\circ$  [Linear pair]

$$\Rightarrow \angle AOC = 180^\circ - 130^\circ = 50^\circ$$

In  $\triangle AOC$ ,  $\angle AOC + \angle ACO + \angle OAC = 180^\circ$

[Angle sum property]

$$\Rightarrow \angle ACO = 180^\circ - 50^\circ - 90^\circ = 40^\circ$$

10.

$$\text{Given, } \angle QPT = 60^\circ$$

$\because$  OP is the radius of the circle

$\therefore \angle OPT = 90^\circ$  ( $\because$  Tangent is perpendicular to the radius through the point of contact.)

$$\Rightarrow \angle OPQ = \angle OPT - \angle QPT = 90^\circ - 60^\circ = 30^\circ$$

$$\text{i.e., } \angle OPQ = \angle OQP = 30^\circ \quad (\because OP = OQ)$$

$$\text{In } \triangle OPQ, \angle POQ = 180^\circ - (30^\circ + 30^\circ) = 120^\circ$$

$$\text{Now, reflex } \angle POQ = 360^\circ - 120^\circ = 240^\circ$$

As, we know that

$$\Rightarrow \angle PRQ = \left( \frac{240}{2} \right)^\circ = 120^\circ$$

11.

Given,  $OP$  bisects the chord  $AD$

$\therefore OP \perp AD$

[ $\because$  The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord]

$\Rightarrow m\angle P = 90^\circ$

Since,  $BC$  is the tangent to the circle at point  $B$ .

Therefore,  $OB$  is perpendicular to  $BC$ .

$\therefore m\angle B = 90^\circ$

Now,  $m\angle BOP + m\angle AOP = 180^\circ$  [Linear pair]

$\Rightarrow m\angle BOP + 60^\circ = 180^\circ$

$\Rightarrow m\angle BOP = 120^\circ$  [ $\because$  Given,  $m\angle AOP = 60^\circ$ ]

In a quadrilateral  $BOPC$ ,

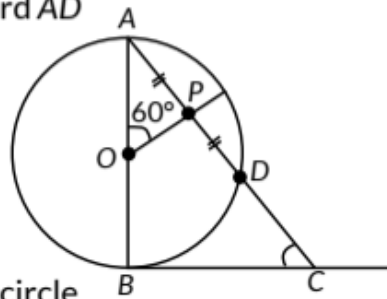
$m\angle B + m\angle BOP + m\angle P + m\angle C = 360^\circ$

[ $\because$  Sum of the angles of a quadrilateral is  $360^\circ$ ]

$\Rightarrow 90^\circ + 120^\circ + 90^\circ + m\angle C = 360^\circ$

$\Rightarrow m\angle C = 360^\circ - 300^\circ = 60^\circ$

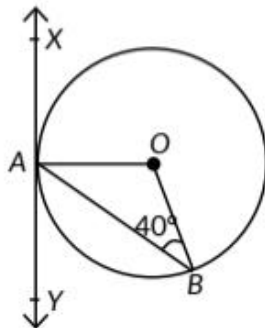
Hence,  $m\angle C = 60^\circ$



12. Since,  $XAY$  is a tangent to the circle centred at  $O$ . Therefore,  $OA$  is perpendicular to  $XAY$

$\therefore m\angle XAO = 90^\circ$

$\therefore m\angle XAO = 90^\circ$



Now,  $OA = OB$  [Radii of circle]

$\Rightarrow m\angle OAB = m\angle OBA$  [Angles opposite to equal sides of a triangle are also equal]

Given,  $m\angle ZABO = 40^\circ$

Therefore,  $m\angle OAB = 40^\circ$

Now,  $m\angle BAY + m\angle ZOAB + m\angle ZXA O = 180^\circ$  [ $ZXAY$  is a straight angle]

$$m\angle BAY + 40^\circ + 90^\circ = 180^\circ$$

$$\Rightarrow m\angle BAY = 180^\circ - 130^\circ = 50^\circ$$

Now, in  $\triangle AOB$ ,

$$m\angle AOB + m\angle OAB + m\angle OBA = 180^\circ \text{ [Angle sum property]}$$

$$\Rightarrow m\angle AOB + 40^\circ + 40^\circ = 180^\circ$$

$$\Rightarrow m\angle AOB + 80^\circ = 180^\circ$$

$$\Rightarrow m\angle AOB = 180^\circ - 80^\circ = 100^\circ$$

13. Since angle subtended by an arc at the centre is double the angle subtended by the same arc at the remaining part of the circle.

$$\therefore 2\angle ABQ = \angle AOQ$$

$$\Rightarrow \angle ABQ = \frac{58^\circ}{2} \Rightarrow \angle ABQ = 29^\circ$$

Also,  $\angle BAT = 90^\circ$  ( $\because$  Tangent is perpendicular to the radius through the point of contact)

$$\text{In } \triangle ABT, \angle ABT + \angle BAT + \angle ATB = 180^\circ$$

$$\Rightarrow 29^\circ + 90^\circ + \angle ATQ = 180^\circ$$

$$\Rightarrow \angle ATQ = 180^\circ - 119^\circ = 61^\circ$$

14.

(a): Given,  $\angle ABO = 30^\circ$ ,  $OA = 75$  cm

$$\text{In } \triangle OAB, \tan 30^\circ = \frac{OA}{AB}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{AB} \Rightarrow AB = 75\sqrt{3} \text{ cm}$$

$$(b) \text{ In } \triangle OAB, \sin 30^\circ = \frac{OA}{OB}$$

$$\Rightarrow \frac{1}{2} = \frac{75}{OB} \Rightarrow OB = 150 \text{ cm}$$

(c) In  $\triangle OAB$ ,  $PQ \parallel OA$

$$\frac{QB}{QO} = \frac{BP}{AP}$$

$$\Rightarrow \frac{150 - 75}{75} = \frac{AB}{AP} - 1 \Rightarrow 2 = \frac{AB}{AP} = \frac{75\sqrt{3}}{AP}$$

$$\Rightarrow AP = 75 \times \frac{\sqrt{3}}{2} \Rightarrow AP = \frac{75\sqrt{3}}{2} \text{ cm}$$

OR

$OA = OQ = 75 \text{ cm}$  ( $\because$  Radius)  
In  $\triangle OAB$ ,  
We have,  $PQ \parallel OA$   
In  $\triangle BQP$  and  $\triangle BOA$   
 $\angle BQP = \angle BOA$  (corresponding angles)  
 $\angle B = \angle B$  (common)  
 $\therefore \triangle BQP \sim \triangle BOA$  (By AA similarity)  
 $\therefore \frac{BQ}{BO} = \frac{QP}{OA} = \frac{BP}{BA}$   
 $\Rightarrow \frac{PQ}{75} = \frac{AB - AP}{AB}$   
 $\Rightarrow \frac{PQ}{75} = 1 - \frac{AP}{AB}$   
 $\Rightarrow \frac{PQ}{75} = 1 - \frac{75\sqrt{3}}{2 \times 75\sqrt{3}}$   
 $\Rightarrow \frac{PQ}{75} = \frac{1}{2} \quad \therefore PQ = \frac{75}{2} = 37.5$   
 $\Rightarrow PQ = 37.5 \text{ cm}$

15. Given: Two circles with centres  $O$  and  $O'$  of radii  $2r$  and  $r$  respectively, touch each other internally at  $A$ .  $AB$  is the chord of bigger circle touches the smaller circle at  $C$ . To prove :  $C$  bisects  $AB$  i.e.,  $AC = CB$  Here, for smaller circle ( $O', r$ )

$$\angle ACO = 90^\circ$$

[ $\because$  Angle in a semicircle is  $90^\circ$ ]

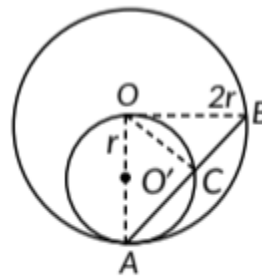
$$\therefore OC \perp AC$$

Now, in bigger circle ( $O, 2r$ )

Since,  $AB$  is a chord and  $OC \perp AB$ .

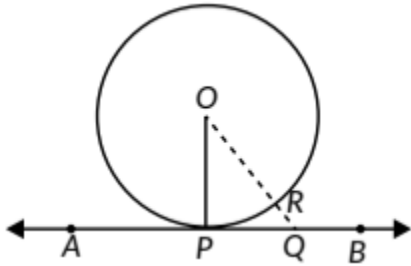
$$\Rightarrow AC = CB \quad [\because \text{Perpendicular drawn from centre of the circle to a chord bisects the chord}]$$

Hence,  $C$  bisects the chord  $AB$ .





16. Given: A circle  $C(O, r)$  and a tangent  $AB$  at a point  $P$ . To prove :  $OP \perp AB$ .  
 Construction:- Take any point  $Q$ , other than  $P$ , on the tangent  $AB$ . Join  $OQ$ .  
 Suppose  $OQ$  meets the circle at  $R$ .



Proof: We know that among all line segments joining the point  $O$  to a point on  $AB$ , the shortest one is perpendicular to  $AB$ . So, to prove that  $OP \perp AB$ , it is sufficient to prove that  $OP$  is shorter than any other segment joining  $O$  to any point of  $AB$ . Clearly,  $OP=OR$  [radii of the same circle]

Now,  $OQ = OR+RQ \Rightarrow OQ > OR$

$\Rightarrow OQ > OP$  [:-  $OP=OR$ ]

$\Rightarrow OP < OQ$

Thus,  $OP$  is shorter than any other segment joining  $O$  to any point of  $AB$ .

Hence,  $OP \perp AB$ .

17. Given Two equal circles  $O$  and  $O'$  touching each other at  $X$ .  $AC$  is tangent to the circle with centre  $O$ .  $\angle ADO' = 90^\circ$

To find:  $\frac{DO'}{CO}$

Solution : Let  $AO' = O'X = XO = r$

$\therefore$  Tangent to a circle is always perpendicular to its radius at the point of contact.

$\therefore \angle ACO = 90^\circ$

In  $\triangle ADO'$  and  $\triangle ACO$

$\angle DAO' = \angle CAO$

[Common]

$\angle ADO' = \angle ACO$

[Each  $90^\circ$ ]

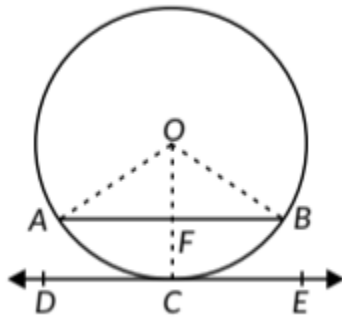
$\therefore \triangle ADO' \sim \triangle ACO$

(AA similarity criteria)

$$\Rightarrow \frac{DO'}{CO} = \frac{AO'}{AO} = \frac{r}{3r} = \frac{1}{3}$$

18. Given: A circle with centre  $O$  and  $C$  is the mid point of arc  $ACB$  and  $DE$  is a tangent to the circle. To prove :  $AB \parallel DE$  Construction: Join  $OA$ ,  $OB$  and  $OC$ .

Proof Since C is the midpoint of arc AB.



$$\therefore \angle AOF = \angle BOF$$

( $\because$  OA and OB are equally inclined with OC)

Now, in  $\triangle OAF$  and  $\triangle OBF$ ,

$$OA = OB$$

(Radii of the circle)

$$\angle AOF = \angle BOF$$

(Proved above)

$$OF = OF$$

(Common)

$$\therefore \triangle OAF \cong \triangle OBF$$

(By SAS criterion)

$$\Rightarrow \angle AFO = \angle BFO$$

(By CPCT)

$$\text{Now, } \angle AFO + \angle BFO = 180^\circ$$

(Linear pair)

$$\Rightarrow 2\angle AFO = 180^\circ \Rightarrow \angle AFO = 90^\circ$$

$$\text{Also, } \angle OCD = 90^\circ$$

Tangent is perpendicular to radius through the point of contact.

$$\therefore \angle AFO = \angle OCD$$

(Each  $90^\circ$ )

But these are corresponding angles  $\therefore AB \parallel DE$

19.

Given :  $O$  is the centre of the circle and  $TP$  is the tangent to circle and  $\angle PBT = 30^\circ$

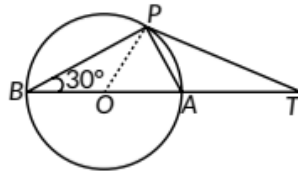
To prove :  $\frac{BA}{AT} = \frac{2}{1}$

Construction : Join  $OP$

Proof :  $\angle BPA = 90^\circ$

(Angle in a semi circle)

In  $\triangle BPA$ ,  $\angle P + \angle PBA + \angle BAP = 180^\circ$



[Angle sum property]

$$\Rightarrow 90^\circ + 30^\circ + \angle BAP = 180^\circ$$

$$\Rightarrow \angle BAP = 60^\circ$$

Also,  $\angle OPT = 90^\circ$

And  $OP = OA$  (Radii of same circle) ... (i)

$$\therefore \angle OAP = \angle OPA = 60^\circ \quad \dots (ii)$$

$$\Rightarrow \angle APT = 90^\circ - 60^\circ = 30^\circ$$

Now,  $\angle OAP + \angle PAT = 180^\circ$  (Linear pair)

$$\Rightarrow 60^\circ + \angle PAT = 180^\circ \quad \text{[Using (ii)]}$$

$$\Rightarrow \angle PAT = 120^\circ$$

In  $\triangle PAT$ ,  $\angle PAT + \angle APT + \angle PTA = 180^\circ$

$$\Rightarrow 120^\circ + 30^\circ + \angle PTA = 180^\circ$$

$$\Rightarrow \angle PTA = 30^\circ$$

$$\Rightarrow PA = AT \quad \dots (iii)$$

Also in  $\triangle OAP$ ,

$$\angle AOP = 180^\circ - 60^\circ - 60^\circ = 60^\circ$$

$$\therefore \angle AOP = \angle OPA \Rightarrow PA = OA \quad \dots (iv)$$

Hence,  $PA = AT = OA = OP$  [Using (i), (iii) and (iv)]

Now,  $BA = BO + OA = 2OA$  ( $\because OA = OB$ )

$$\Rightarrow BA = 2AT \quad (\because OA = AT)$$

$$\Rightarrow \frac{BA}{AT} = \frac{2}{1}$$

20.

(b): Join  $OR$ .

We know that, tangent to a circle is  $\perp$  to radius at the point of contact. So,  $OQ \perp PQ$  and  $OR \perp PR$ .

Also,  $\angle QPR = 90^\circ$

Now, in quadrilateral  $OQPR$ ,

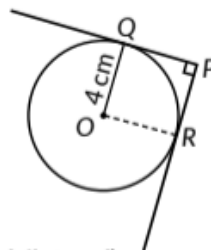
$$\angle QOR = 360^\circ - (90^\circ + 90^\circ + 90^\circ)$$

$$= 90^\circ \quad (\text{By angle sum property of a quadrilateral})$$

Now,  $\angle PQO = \angle QOR = \angle ORP = \angle RPQ = 90^\circ$

Also,  $PQ = PR$

( $\because$  Tangents drawn from an external point are equal)



∴ PQOR is a square.  
Hence, PQ = OQ = 4 cm

21. (a): In quadrilateral AOBP  
 $\angle AOB + \angle OBP + \angle APB + \angle OAP = 360^\circ$   
 $\Rightarrow \angle AOB + 90^\circ + 90^\circ + 80^\circ = 360^\circ$  [∵ OA ⊥ PA and OB ⊥ PB]  
 $\Rightarrow \angle AOB = 360^\circ - 260^\circ = 100^\circ$

22. (b): It is known that the length of the tangents drawn from an external point to a circle are equal.  
 ∴ QP = PT = 3.8 cm and PR = PT = 3.8 cm  
 Now, QR = QP + PR = 3.8 cm + 3.8 cm = 7.6 cm

23. (b): Given,  $\angle QPR = 46^\circ$   
 We have, OQ ⊥ PQ and OR ⊥ RP [∵ Radius is to the tangent through the point of contact]  
 $\angle OQP = \angle ORP = 90^\circ$   
 In quadrilateral PQOR, we have  
 $\angle OQP + \angle QPR + \angle PRO + \angle ROQ = 360^\circ$   
 $= 90^\circ + 46^\circ + 90^\circ + \angle ROQ = 360^\circ$   
 $\Rightarrow \angle ROQ = 360^\circ - 226^\circ = 134^\circ$

24.

(d) : Let common tangent at P meet the tangent AB at R.

Since, tangents drawn from an external point to a circle are equal.

∴ AR = RP

and BR = RP

⇒  $\angle RAP = \angle RPA = x$  (say) ... (i)

and  $\angle RBP = \angle RPB = y$  (say) ... (ii)

Now,  $\angle ARP + \angle BRP = 180^\circ$  [Linear pair] ... (\*)

In  $\triangle ARP$ ,  $\angle ARP + \angle RPA + \angle RAP = 180^\circ$  ... (iii)

and in  $\triangle BRP$ ,  $\angle BRP + \angle RPB + \angle RBP = 180^\circ$  ... (iv)

Adding (iii) and (iv), we get

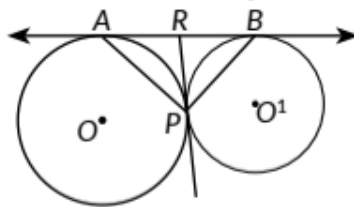
$\angle ARP + x + x + \angle BRP + y + y = 360^\circ$  [Using (i) & (ii)]

⇒  $\angle ARP + \angle BRP + 2x + 2y = 360^\circ$

⇒  $2(x + y) = 360^\circ - 180^\circ = 180^\circ$  [Using (\*)]

⇒  $x + y = 90^\circ$

i.e.,  $\angle RPA + \angle RPB = 90^\circ \Rightarrow \angle APB = 90^\circ$



25. (b): We know, tangents drawn from an external point to the circle are equal in length.

i.e.,  $AP = AS$ ,  $BP = BQ$ ,  $CQ = CR$ ,  $DR = DS$

So,  $CR = CQ = 3$  cm

Now,  $BC = 7$  cm  $\Rightarrow CQ + BQ = 7$  cm

$\Rightarrow BQ = (7 - 3)$  cm = 4 cm

Also,  $BQ = BP \Rightarrow BP = 4$  cm

Also,  $AS = AP = 5$  cm  $\Rightarrow AP = 5$  cm

$AB = AP + PB = (5 + 4)$  cm = 9 cm

So,  $x = 9$

26. (c): Let chord  $AB$  of larger circle is a tangent to the smaller circle.

$\therefore OC \perp AB$

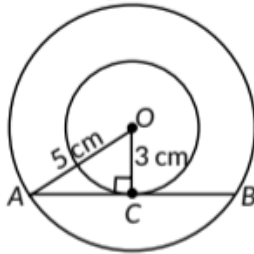
In right  $\triangle OAC$ ,

$$(OA)^2 = (OC)^2 + (AC)^2$$

$$\Rightarrow (5)^2 = (3)^2 + (AC)^2$$

$$\Rightarrow AC^2 = 25 - 9 = 16$$

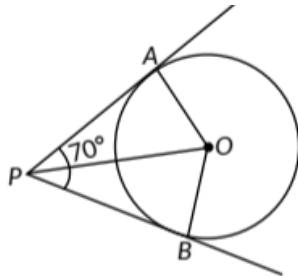
$$\Rightarrow AC = 4$$
 cm



As, perpendicular drawn from the centre to the chord bisects the chord.

$\therefore$  Length of chord  $AB = (2 \times 4)$  cm = 8 cm

27. Given  $\angle APB = 70^\circ$ ,  $PA$  and  $PB$  are tangents to the circle.



$$\angle PAO = \angle PBO = 90^\circ \quad (\because OA \perp PA \text{ and } OB \perp PB)$$

In quadrilateral  $APBO$

$$\angle AOB + \angle APB + \angle PBO + \angle PAO = 360^\circ$$

$$\Rightarrow \angle AOB + 90^\circ + 90^\circ + 70^\circ = 360^\circ$$

$$\Rightarrow \angle AOB = 360^\circ - 250^\circ = 110^\circ$$

In  $\triangle PAO$  and  $\triangle PBO$

$PA = PB$  (Tangents drawn from an external point to the circle are equal)

$$OA = OB \quad (\text{Radii of circle})$$

$$OP = OP \quad (\text{common side})$$

$$\therefore \triangle PAO \cong \triangle PBO \quad (\text{SSS})$$

$$\angle AOP = \angle BOP \quad (\text{CPCT})$$

Now,  $\angle AOB = \angle POA + \angle POB$

$$\Rightarrow \angle AOB = \angle POA + \angle POA = 2\angle POA$$

$$\Rightarrow \angle POA = \frac{1}{2} \times 110^\circ = 55^\circ$$

28. Lengths of tangents drawn from an external point to the circle are equal.

∴ AP = AR = 4 cm, BP = BQ = 3 cm and CQ = CR = 7 cm

∴ BC = BQ + CQ = 3 + 7 = 10 cm

29. Since, tangents drawn from an external point are equally inclined to the line joining centre to that point.

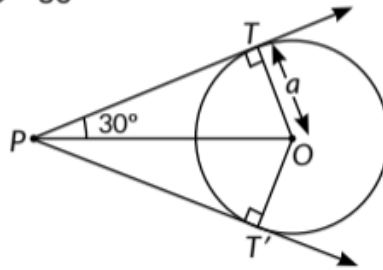
$$\therefore \angle TPT' = 60^\circ \Rightarrow \angle TPO = 30^\circ$$

Also,  $OT \perp TP$

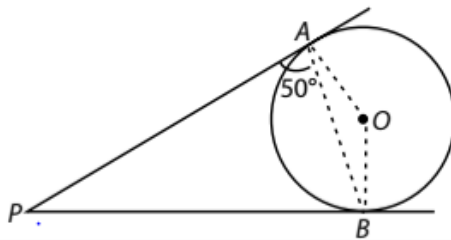
Now, in  $\triangle TPO$ ,  $\sin 30^\circ = \frac{OT}{OP}$

$$\Rightarrow \frac{1}{2} = \frac{a}{OP}$$

$$\Rightarrow OP = 2a$$



30. Given, PA and PB are tangents to a circle and  $\angle PAB = 50^\circ$



Since, tangent at any point of a circle is perpendicular to the radius through the point of contact.

i.e.  $OA \perp PA$  and  $OB \perp PB \Rightarrow \angle OAP = \angle OBP = 90^\circ$

$$\therefore \angle OAB = \angle OAP - \angle PAB$$

$$= 90^\circ - 50^\circ = 40^\circ = \angle OBA \quad (\because OA = OB)$$

In  $\triangle AOB$ ,  $\angle AOB = 180^\circ - 40^\circ - 40^\circ = 100^\circ$

31.  $PA = PB$

(Tangents drawn from external point are equal)

$$\Rightarrow \angle ABP = \angle BAP = x \text{ (say)}$$

(∵ Angles opposite to equal sides are equal)

In  $\triangle APB$ ,

$$50^\circ + x + x = 180^\circ$$

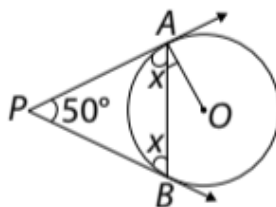
$$\Rightarrow 2x = 130^\circ$$

$$\Rightarrow x = 65^\circ$$

Now,  $\angle OAP = 90^\circ$

(∵ Tangent is perpendicular to the radius through the point of contact)

$$\therefore \angle OAB = 90^\circ - 65^\circ = 25^\circ$$



32. Let  $O$  be the centre of concentric circles and  $AB$  be the chord for circle  $C_1$ , and tangent for circle  $C_2$ .

Let  $P$  be the point where  $AB$  meets  $C_2$ .

Join  $O$  to  $A$  and  $O$  to  $P$ .

$\therefore$  Tangent at any point is perpendicular to the radius through the point of contact.

$$\therefore \angle OPA = 90^\circ$$

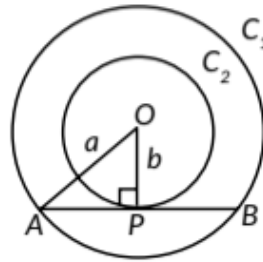
Now, in  $\triangle OPA$ ,  $OA^2 = OP^2 + AP^2$

$$\Rightarrow a^2 = b^2 + AP^2$$

$$[\because OA = a, OP = b] \Rightarrow AP = \sqrt{a^2 - b^2}$$

We know that,  $AB = 2AP$

$$\Rightarrow AB = 2\sqrt{a^2 - b^2}.$$



33.

**Quest** To prove: -  $ORPQ$  is a square i.e.  $\angle O = \angle Q = \angle P = \angle R = 90^\circ$   
and  $OQ = QR = PR = OR$

**Proof:** -  $\angle OQP = 90^\circ$   
 $\angle ORP = 90^\circ$  } Tangent is perpendicular to the radius at the point of contact. — (1)

Now, In  $\triangle ORP$  and  $\triangle OQP$   
 $OQ = OR$  (Common)  
 $OR = OQ$  (Radii of same circle)  
 $PR = PQ$  (Tangents from an external point to a circle are equal)  
 Therefore,  $\triangle ORP \cong \triangle OQP$  by SSS rule  
 $\therefore \angle OPR = \angle OPQ$  (By CPCT)

$\Rightarrow \angle OPR = \angle OPQ = 45^\circ$   
 $\Rightarrow \angle OPR + \angle OPQ = 90^\circ$   
 $\Rightarrow \angle QPR = 90^\circ$  — (2)

In quad  $OQPR$   
 $\angle OQP + \angle QPR + \angle PRO + \angle ROQ = 360^\circ$  (Angle sum property)  
 $90 + 90 + 90 + \angle ROQ = 360$   
 $\angle ROQ = 360 - 270$   
 $\angle ROQ = 90^\circ$  — (3)

Also  $OR = PR$  — (4)  
 (As  $\angle ROQ = \angle PRO = 45^\circ$ ) (Isosceles triangle property)  
 Also  $OQ = PR$  (Tangents from an external point to a circle are equal) — (5)

$\Rightarrow$  Using (4) and (5)  
 $OQ = PQ = PR = OR$  — (6)

Using (1), (2), (3), (6)  
 $ORPQ$  is a square

Hence proved.

**[Topper's Answer, 2022]**

34. Given,  $AQ = 5$  cm

$AQ = AR = 5$  cm (Tangents drawn from an external point to the circle are equal)

Now,  $AC = AR + RC$  (OR is a perpendicular bisector of AC ..  $AR = RC$ )

$\Rightarrow AC = 10$  cm



35. Given: PQ and PR are tangents from an external point P. To prove : PQOR is a cyclic quadrilateral. Proof OR and OQ are the radius of circle centred at O, and PR and PQ are tangents.

$$\therefore \angle ORP = 90^\circ \text{ and } \angle OQP = 90^\circ$$

In quadrilateral PQOR, we have

$$\angle P + \angle R + \angle O + \angle Q = 360^\circ$$

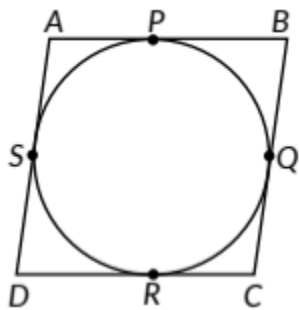
$$\Rightarrow \angle P + 90^\circ + \angle O + 90^\circ = 360^\circ$$

$$\Rightarrow \angle P + \angle O = 360^\circ - 180^\circ = 180^\circ$$

$\angle P$  and  $\angle O$  are opposite angles of quadrilateral which are supplementary.

$\therefore$  PQOR is a cyclic quadrilateral.

36. Let the circle touches the sides AB, BC, CD and DA of quadrilateral ABCD at P, Q, R and S respectively. Since, lengths of tangents drawn from an external point to the circle are equal.



$$\therefore AP = AS \quad \dots(1) \quad (\text{Tangents drawn from } A)$$

$$BP = BQ \quad \dots(2) \quad (\text{Tangents drawn from } B)$$

$$CR = CQ \quad \dots(3) \quad (\text{Tangents drawn from } C)$$

$$DR = DS \quad \dots(4) \quad (\text{Tangents drawn from } D)$$

Adding (1), (2), (3) and (4), we get

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow (AP + PB) + (CR + RD) = (AS + SD) + (BQ + QC)$$

$$\Rightarrow AB + CD = AD + BC$$

37. As we know that, tangents drawn from an external point are equal in length.

$$\therefore BP = BQ \text{ and } CD = CQ \dots(i)$$

Also,  $AP = AQ = 12 \text{ cm}$

$$\Rightarrow AB + BP = 12 \text{ cm and } AC + CQ = 12 \text{ cm}$$

$$\Rightarrow AB + BQ = 12 \text{ cm}$$

$$AC + CD = 12 \text{ cm} \dots (ii) \text{ [Using (i)]}$$

Now, perimeter of  $\triangle ABC = AB + BC + CA$

$$\begin{aligned}
&= AB + BD + DC + AC \\
&= 12 + 12 \text{ [Using (ii)]} \\
&= 24 \text{ cm}
\end{aligned}$$

38. Since, tangents drawn from an external point are equal.

$$\therefore TP = TQ$$

$$= \angle TPQ = \angle TQP \dots (i) \text{ (:- Angles opposite to equal sides are equal)}$$

In  $\triangle TPQ$ ,

$$\angle PTQ + \angle TQP + \angle TPQ = 180^\circ$$

$$= \angle PTQ + \angle TPQ + \angle TPQ = 180^\circ \text{ [Using (i)]}$$

$$= \angle PTQ + 2 \angle TPQ = 180^\circ$$

$$= \angle PTQ = 180^\circ - 2 \angle TPQ \dots (ii)$$

Now,  $\angle OPT = 90^\circ$  (:- Tangent is perpendicular to the radius through the point of contact)

$$\therefore \angle TPQ = 90^\circ - \angle OPQ \dots (iii)$$

$$\text{From (ii) and (iii), } \angle PTQ = 180^\circ - 2(90^\circ - \angle OPQ)$$

$$= 180^\circ - 180^\circ + 2 \angle OPQ = 2 \angle OPQ$$

39. As we know that, tangents drawn from an external point are equal in length.

$$\therefore QC = CA; QD = BD \text{ and } PA = PB$$

Since  $QC = QD = 3 \text{ cm}$  (given)

$$= CA = BD = 3 \text{ cm}$$

$$\text{Also, } PC = PA - AC$$

$$= PC = (12 - 3) \text{ cm} = 9 \text{ cm}$$

Similarly  $PD = 9 \text{ cm}$

$$\therefore PC + PD = 9 + 9 = 18 \text{ cm}$$

40. Given: Tangents  $PA$  and  $PB$  are drawn at end points of a chord  $AB$  of a circle  $C(O, r)$ .

To prove: Tangents  $PA$  and  $PB$  make equal angles with the chord  $AB$  i.e.,

$$\angle PAB = \angle PBA$$

Proof: Since, lengths of tangents drawn from an external point to the circle are equal.

$$\therefore PA = PB$$

$$= \angle PAB = \angle PBA \text{ (:- Angles opposite to equal sides are equal)}$$

41. Let  $AD = x \Rightarrow AF = x$

[∵ Tangents drawn from an external point to the circle are equal in lengths]

∴  $BD = BE = 12 - x$  (...  $AB = 12$  cm)

and  $FC = CE = 10 - x$  (...  $AC = 10$  cm)

But  $BC = 8$  cm (Given)

∴  $10 - x + 12 - x = 8$

$= -2x + 22 = 8 \Rightarrow 2x = 14 \Rightarrow x = 7$

Hence, length of  $AD = 7$  cm

Length of  $BE = 12 - 7 = 5$  cm

Length of  $CF = 10 - 7 = 3$  cm

42. Given,  $PA$  and  $PB$  are tangents from an external point  $P$ .

$PA = PB = 5$  cm

$\angle PAB = \angle PBA$

(∵ Angles opposite to equal sides are equal)

In  $\triangle APB$ , by angle sum property

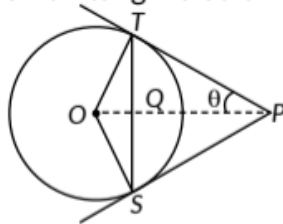
$\angle APB + \angle PAB + \angle PBA = 180^\circ$

$\angle APB + \angle PAB + \angle PAB = 180^\circ$

∴  $AB = PB = PA = 5$  cm

43.

In  $\triangle OTP$ ,  $OT = r$ ,  $OP = 2r$  [Given]  
 $\angle OTP = 90^\circ$  [Radius is perpendicular to tangent at the point of contact]  
 Let  $\angle TPO = \theta$   
 $\therefore \sin \theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2}$   
 $\Rightarrow \theta = 30^\circ$   
 $\therefore$  In  $\triangle TOP$ ,  $\angle TOP = 60^\circ$   
 [By angle sum property]  
 $\angle TOP = \angle SOP$  [As  $\triangle$ 's are congruent]  
 $\Rightarrow \angle SOP = 60^\circ \therefore \angle TOS = 120^\circ$   
 In  $\triangle OTS$ , as  $OT = OS$  (Radii of same circle)  
 $\therefore \angle OST = \angle OTS$



Now,  $\angle OTS + \angle OST + \angle TOS = 180^\circ$

$\Rightarrow 2\angle OST + 120^\circ = 180^\circ \therefore \angle OTS = \angle OST = 30^\circ$

44. Given,  $\angle SPT = 120^\circ$

Since, radius is perpendicular to the tangent at the point of contact.

∴  $\angle OSP = 90^\circ$  Also,  $\angle SPO = 60^\circ$  [∵ Tangents drawn to a circle from an external

point are equally inclined to the segment, joining the centre to that point]

$$\text{In } \Delta SPO, \cos 60^\circ = \frac{PS}{OP} \Rightarrow \frac{1}{2} = \frac{PS}{OP} \Rightarrow OP = 2PS$$

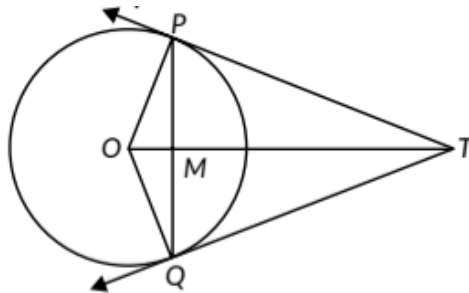
45. In right AAOP,  $OP^2 = AP^2 + OA^2 = 8^2 + 6^2 = 100$

In right ABOP,  $OP^2 = BP^2 + OB^2$

$$\Rightarrow 100 = BP^2 + 4^2$$

$$\Rightarrow BP^2 = 100 - 16 = 84 \Rightarrow BP = 2\sqrt{21} \text{ cm}$$

46. Given: A circle with centre O, two tangents TP and TQ drawn from external point T to the circle.



To prove : OT is right bisector of PQ i.e.,

$$\angle TMP = 90^\circ, PM = MQ$$

Proof : In  $\Delta TPM$  and  $\Delta TQM$ , we have

$$TP = TQ \quad (\because \text{Tangents drawn from an external point are equal})$$

$$TM = TM \quad (\text{Common})$$

$$\angle MTP = \angle MTQ \quad [\because TP \text{ and } TQ \text{ are equally inclined to } OT]$$

$$\therefore \Delta TPM \cong \Delta TQM \quad (\text{By SAS congruency})$$

$$\Rightarrow PM = MQ \quad (\text{By CPCT}) \dots(i)$$

$$\text{Also, } \angle TMP = \angle TMQ \quad (\text{By CPCT}) \dots(ii)$$

$$\text{But } \angle TMP + \angle TMQ = 180^\circ \quad [\text{Linear Pair}]$$

$$\Rightarrow 2 \angle TMP = 180^\circ \Rightarrow \angle TMP = 90^\circ$$

Hence, OT is the right bisector of line segment PQ.

47. Given: Two tangents PR and QR are drawn from an external point R and  $\angle PRQ = 120^\circ$

To prove :  $OR = PR + RQ$ .

Construction: Join OP and OQ.

Proof: In  $\Delta OPR$  and  $\Delta OQR$ ,

$$OP = OQ \quad (\text{Radii of the circle})$$

$$\angle OPR = \angle OQR = 90^\circ \quad (\because \text{Radius is perpendicular to the tangent at the point of contact})$$

$$PR = RQ \quad (\text{Tangents from an external point are equal})$$

$$\begin{aligned} \therefore \triangle OPR &\cong \triangle OQR \\ \text{So, } \angle PRO &= \angle QRO \\ \therefore \angle PRO &= \frac{1}{2} \angle PRQ = \frac{1}{2} \times 120^\circ = 60^\circ \end{aligned}$$

In  $\triangle OPR$ , we have

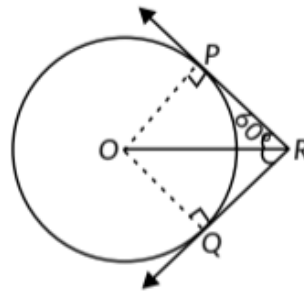
$$\cos 60^\circ = \frac{PR}{OR}$$

$$\Rightarrow \frac{1}{2} = \frac{PR}{OR}$$

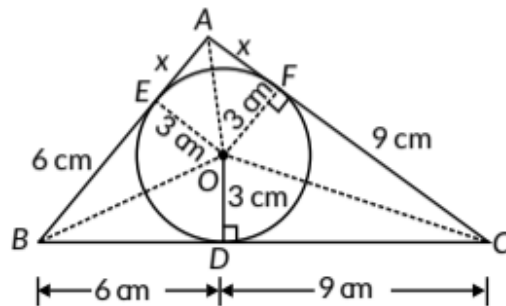
$$\Rightarrow OR = 2PR \Rightarrow OR = PR + PR$$

$$\Rightarrow OR = PR + RQ (\because PR = RQ)$$

(By SAS)  
(By CPCT)



48. Let E and F be the points where the tangents AB and AC touches the circle respectively. Join OE and OF. Now, radius is perpendicular to tangent at the point of contact.



So,  $OD \perp BC$ ,  $OE \perp AB$  and  $OF \perp AC$ .

Join OA, OB and OC.

Since, tangents drawn from an external point to a circle are equal.

$\therefore BD = BE = 6$  cm,  $CD = CF = 9$  cm and  $AE = AF = x$  cm (say)

Now, area of  $\triangle ABC =$  area of  $\triangle AOB +$  area of  $\triangle BOC +$  area of  $\triangle AOC$

$$\Rightarrow 54 = \frac{1}{2} \times AB \times OE + \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AC \times OF$$

$$\Rightarrow 54 = \frac{1}{2} \times (x+6) \times 3 + \frac{1}{2} \times 15 \times 3 + \frac{1}{2} \times (x+9) \times 3$$

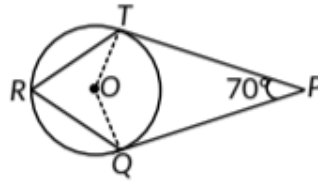
$$\Rightarrow 54 = \frac{1}{2} \times (3x+18+45+3x+27)$$

$$\Rightarrow 6x + 90 = 108 \Rightarrow 6x = 108 - 90 = 18 \Rightarrow x = \frac{18}{6} = 3$$

$$\therefore AB = 3 + 6 = 9 \text{ cm and } AC = 3 + 9 = 12 \text{ cm}$$

49.

Given :  $O$  is the centre of a circle.  $PT$  and  $PQ$  are tangents to the circle from an external point  $P$ .  
 $\angle TPQ = 70^\circ$



Construction : Join  $T$  to  $O$  and  $Q$  to  $O$

$\therefore OT \perp PT$  and  $OQ \perp QP \therefore \angle OTP = \angle OQP = 90^\circ$

In quadrilateral  $PTOQ$ ,

$\angle OTP + \angle OQP + \angle TOQ + \angle TPQ = 360^\circ$

$\Rightarrow 90^\circ + 90^\circ + \angle TOQ + 70^\circ = 360^\circ$

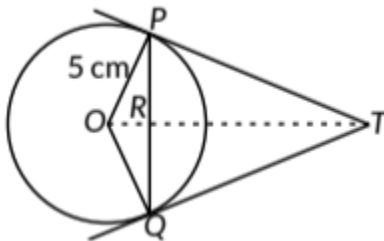
$\Rightarrow \angle TOQ + 70^\circ = 180^\circ \Rightarrow \angle TOQ = 110^\circ$

Also,  $\angle TOQ = 2\angle TRQ$

[ $\because$  Angle subtended by an arc at centre of the circle is double the angle subtended by the same arc at the remaining part of the circle]

$\therefore 110^\circ = 2\angle TRQ \Rightarrow \angle TRQ = \frac{110^\circ}{2} = 55^\circ$

50. Join  $OT$  and let  $PQ$  intersect  $OT$  at  $R$ . We know that, tangents drawn to a circle from an external point are equally inclined to the segment, joining the centre to that point.



So,  $\angle OTP = \angle OTQ$

In  $\triangle TPR$  and  $\triangle TQR$ ,

$TP = TQ$  (Tangents drawn from an external point are equal in length)

$\angle RTP = \angle RTQ$  ( $\because \angle OTP = \angle OTQ$ )

$RT = RT$  (common)

$\therefore \triangle TPR \cong \triangle TQR$  (SAS congruence)

Now,  $PR = QR$  (By CPCT)

Also,  $OR \perp PQ$  or  $\angle ORP = \angle ORQ = 90^\circ$

Now,  $PQ = PR + QR$

$\Rightarrow PQ = PR + PR$  [ $\because PR = QR$ ]

$\Rightarrow PR = 4 \text{ cm}$

Now, in  $\triangle OPR$ ,  $OR^2 + PR^2 = OP^2$

$$\Rightarrow OR^2 = OP^2 - PR^2$$

$$\Rightarrow OR^2 = 5^2 - 4^2 \Rightarrow OR = 3 \text{ cm}$$

Let  $RT = x$

$$\text{In } \triangle PRT, PT^2 = PR^2 + RT^2 = 4^2 + x^2 = 16 + x^2$$

$$\text{In } \triangle OPT, PT^2 = OT^2 - OP^2$$

$$\Rightarrow 16 + x^2 = (3 + x)^2 - 5^2 \Rightarrow 16 + x^2 = 9 + 6x + x^2 - 25$$

$$\Rightarrow 6x = 32 \Rightarrow x = \frac{32}{6} = \frac{16}{3}$$

$$\therefore PT^2 = 16 + \left(\frac{16}{3}\right)^2 \Rightarrow PT^2 = 16 + \frac{256}{9}$$

$$\Rightarrow PT^2 = \frac{144 + 256}{9} = \frac{400}{9} \Rightarrow PT = \sqrt{\frac{400}{9}} = \frac{20}{3} \text{ cm}$$

$$\text{So, } TP = TQ = \frac{20}{3} \text{ cm}$$

51. Let  $XY$  and  $PQ$  be two parallel tangents to a circle with centre  $O$ .

Construction : Join  $OB$  and  $OC$ .

Draw  $OA \parallel XY \parallel PQ$ .

Now,  $XB \parallel AO$

$$\Rightarrow \angle XBO + \angle AOB = 180^\circ$$

Now,  $\angle XBO = 90^\circ$  ( $\because$  Tangent to a circle is perpendicular to the radius through the point of contact)

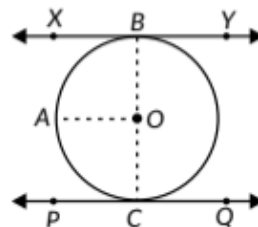
$$\therefore 90^\circ + \angle AOB = 180^\circ \Rightarrow \angle AOB = 180^\circ - 90^\circ = 90^\circ$$

Similarly,  $\angle AOC = 90^\circ$

$$\therefore \angle AOB + \angle AOC = 90^\circ + 90^\circ = 180^\circ$$

Hence,  $BOC$  is a straight line passing through  $O$ .

Thus, the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.



52. Since, we know that tangents drawn from an external point are equal in length.

$$\therefore AE = EC \text{ and } ED = EB \dots(i)$$

$$\text{Now, } AB = AE + EB \dots(ii)$$

Using (i) in (ii), we get  $AB = CE + ED = CD$   
 $\Rightarrow AB = CD$

53. Since, tangents drawn from an external point are equal in length

$\therefore AF = AE, BF = BD, CD = CE$

and  $AB = AC$  (given) ... (i)

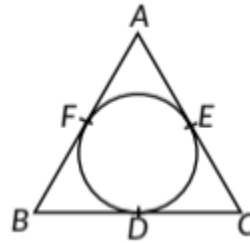
Now, subtracting  $AF$  on both the sides of (i)

$$AB - AF = AC - AF$$

$$= AB - AF - AC + AE \quad [ \because AF = AE ]$$

$$= BF = EC$$

$$= BD = CD \quad [ \because BF = BD, EC = CD ]$$



54. Since lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore XP = XQ$  ... (i),  $AP = AR$  ... (ii),  $BQ = BR$  ... (iii)

From (i),  $XP = XQ$

$$= XA + AP = XB + BQ$$

$$= XA + AR = XB + BR \quad [ \text{Using (ii) and (iii)} ]$$

55. Since, tangents drawn from an external point are equal.

$\therefore TP = TQ$

$+ \angle TPQ = \angle TQP$  ... (i)

(. Angles opposite to equal sides are equal)

In  $\triangle TPQ$ ,

$$\angle PTQ + \angle TQP + \angle TPQ = 180^\circ$$

$$\Rightarrow \angle PTQ + \angle TPQ + \angle TPQ = 180^\circ \quad [ \text{Using (i)} ]$$

$$\Rightarrow \angle PTQ + 2 \angle TPQ = 180^\circ$$

$$\Rightarrow \angle PTQ = 180^\circ - 2 \angle TPQ \quad \dots (ii)$$

Now,  $\angle OPT = 90^\circ$  ( $\because$  Tangent is perpendicular to the radius through the point of contact)

$\therefore \angle TPQ = 90^\circ - \angle OPQ$  ... (iii)

$$\text{From (ii) and (iii), } \angle PTQ = 180^\circ - 2(90^\circ - \angle OPQ)$$

$$= 180^\circ - 180^\circ + 2 \angle OPQ = 2 \angle OPQ$$

56. Given,  $\angle B = 90^\circ$ ,  $AD = 17$  cm,  $AB = 20$  cm,  $DS = 3$  cm Now,  $DS = DR$  and  $AR = AQ$

[. Tangents drawn from an external point to the circle are equal]

$\therefore DR = 3$  cm



$$AR = AD - DR = 17 - 3 = 14 \text{ cm}$$

$$\therefore AQ = 14 \text{ cm}$$

$$\text{Now, } BQ = AB - AQ = 20 - 14 = 6 \text{ cm}$$

OQ  $\perp$  BQ, OP  $\perp$  BP ( $\because$  Tangent at any point of a circle is perpendicular to the radius through the point of contact)

$\therefore$  Quadrilateral BQOP is a square

$$\therefore BQ = OQ = r = 6 \text{ cm}$$

Hence, the radius of the circle = 6 cm.

So,  $\triangle OAP \cong \triangle OBP$  [By SSS congruency criterion]

So,  $\angle APO = \angle BPO$

Hence, OP bisects  $\angle APB$

58. Let the centre of the two concentric circles is O and AB be the chord of the larger circle which touches the smaller circle at point P as shown in figure.

$\therefore$  AB is a tangent to the smaller circle at point P

$$\Rightarrow OP \perp AB$$

By Pythagoras theorem, in  $\triangle OPA$

$$OA^2 = AP^2 + OP^2$$

$$\Rightarrow 5^2 = AP^2 + 3^2$$

$$\Rightarrow AP^2 = 5^2 - 3^2 = 25 - 9$$

$$\Rightarrow AP^2 = 16 \Rightarrow AP = 4 \text{ cm}$$

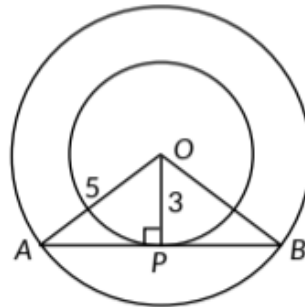
In  $\triangle OPB$

Since,  $OP \perp AB$

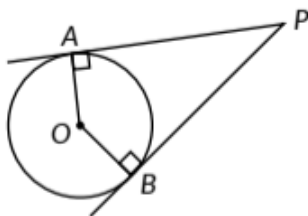
$AP = PB$  ( $\because$  Perpendicular drawn from the centre of the circle bisects the chord)

$$\therefore AB = 2AP = 2 \times 4 = 8 \text{ cm}$$

$\therefore$  The length of the chord of the larger circle is 8 cm.



59. Let PA and PB are two tangents on a circle from point P as shown in the figure. It is known that tangent to a circle is perpendicular to the radius through the point of contact.



$$\therefore \angle ZOAP = \angle ZOBP = 90^\circ \dots (i)$$

In quadrilateral AOBP,

$$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$$

$$= 90^\circ + \angle APB + 90^\circ + \angle BOA = 360^\circ \text{ [Using (i)]}$$

$$= \angle APB + \angle BOA = 180^\circ \text{ Hence proved}$$

60. Given TP and SP are tangents from an external point P.

$\therefore TP = PS = 4 \text{ cm}$  ( $\therefore$  Tangents drawn from an external point to the circle are equal)

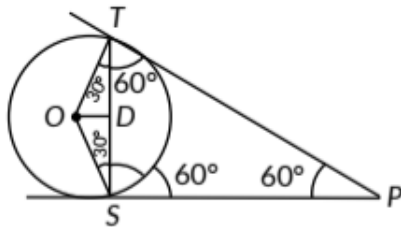
$$\angle TPS = \angle ZPST$$

( $\therefore$  Angles opposite to equal sides are equal)

In  $\triangle TPS$ , by angle sum property

$$\angle TPS = \angle PTS = \angle PST = 60^\circ$$

$\Rightarrow \triangle TPS$  is an equilateral triangle.



$$\therefore TP = PS = TS = 4 \text{ cm}$$

$$\angle OSP = 90^\circ \text{ and } \angle TSP = 60^\circ$$

$$\therefore \angle OSD = 30^\circ$$

$$\text{Now, } \frac{DS}{OS} = \cos 30^\circ \Rightarrow \frac{2}{OS} = \frac{\sqrt{3}}{2} \Rightarrow OS = \frac{4\sqrt{3}}{3} \text{ cm}$$

61. Let  $BL = x \Rightarrow BN = x$

[ $\therefore$  Tangents drawn from an external point to the circle are equal in length]

$$\therefore CL = CM = 8 - x \text{ [}\therefore BC = 8 \text{ cm]}$$

$$\text{and } AN = AM = 10 - x \text{ [}\therefore AB = 10 \text{ cm]}$$

But  $AC = 12 \text{ cm}$  [Given]

$$\therefore AM + MC = 12$$

$$= 10 - x + 8 - x = 12$$

$$= 18 - 2x = 12 \Rightarrow 2x = 6 \Rightarrow x = 3$$

Length of  $BL = 3 \text{ cm}$

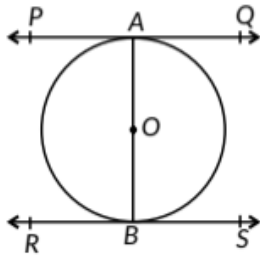
Length of CM =  $8-3=5$ cm

Length of AN =  $10-3=7$ cm

62. Given: A circle  $C(O, r)$  with diameter AB and let PQ and RS be the tangents drawn to the circle at point A and B. To

prove :  $PQ \parallel RS$

Proof : Since tangent at a point to a circle is perpendicular to the radius through the point of contact.



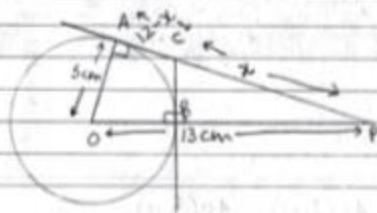
$\therefore AB \perp PQ$  and  $AB \perp RS$

$\Rightarrow \angle PAB = 90^\circ$  and  $\angle ABS = 90^\circ \Rightarrow \angle PAB = \angle ABS$

$\Rightarrow PQ \parallel RS$  [ $\because \angle PAB$  and  $\angle ABS$  are alternate interior angles]

63.

Ques 12.



$\angle OAP = 90^\circ$  (Tangent is perpendicular to radius at the point of contact)

Using pythagoras theorem in  $\triangle OAP$

$$H^2 = B^2 + P^2$$

$$OP^2 = AP^2 + OA^2 \Rightarrow AP^2 = 144$$

$$13^2 = AP^2 + 5^2$$

$$169 = AP^2 + 25$$

$$169 - 25 = AP^2$$

$$AP = \sqrt{144} = 12 \text{ cm}$$

Let PC be x and AC be 12-x

$AC = BC = 12 - x$  (Tangents from point C to the circle are equal)

Also  $OP = 13 \text{ cm}$

$$OB = 5 \text{ cm}$$

$$BP = OP - OB$$

$$BP = 8 \text{ cm}$$

$\angle OBC = 90^\circ$  (Tangent is perpendicular to the radius at the POC)

$\Rightarrow \angle CBP = 90^\circ$  (Linear pair)

Using pythagoras theorem in  $\triangle CBP$

$$H^2 = B^2 + P^2$$

$$CP^2 = BP^2 + BC^2 \quad BC = 12 - x$$

$$x^2 = 8^2 + (12 - x)^2 \quad BC = 12 - \frac{26}{3}$$

$$x^2 = 64 + 144 + x^2 - 24x$$

$$24x = 64 + 144$$

$$24x = 208$$

$$x = \frac{208}{24} = \frac{104}{12} = \frac{26}{3}$$

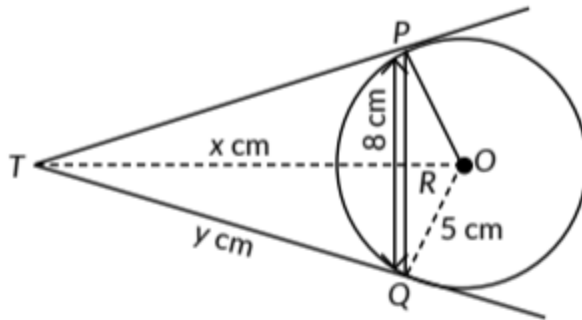
$$BC = \frac{36 - 26}{3} = \frac{10}{3} = 3.34 \text{ m}$$

$$\begin{array}{r} 144 \\ + 25 \\ \hline 169 \\ \sqrt{\phantom{169}} \\ 144 \\ \hline 25 \\ \sqrt{\phantom{25}} \\ 25 \\ \hline 0 \end{array}$$

[Topper's Answer, 2022]



64. We have,  $PQ = 8 \text{ cm}$ ,  $OQ = 5 \text{ cm}$   
 $\therefore PR = RQ = 4 \text{ cm}$



In  $\triangle ORQ$ , by Pythagoras Theorem, we have

$$\begin{aligned} OQ^2 &= OR^2 + RQ^2 \\ \Rightarrow 5^2 &= OR^2 + 4^2 \Rightarrow 25 = OR^2 + 16 \\ \Rightarrow OR^2 &= 25 - 16 = 9 = 3^2 \\ \Rightarrow OR &= 3 \text{ cm} \end{aligned}$$

Let  $TR = x \text{ cm}$  and  $TQ = y \text{ cm}$

Then,  $OT = (x + 3) \text{ cm}$

$$\text{In } \triangle TRQ, TQ^2 = TR^2 + RQ^2 \quad [\text{By Pythagoras Theorem}]$$

$$\Rightarrow y^2 = x^2 + 4^2$$

$$\Rightarrow y^2 = x^2 + 16 \quad \dots(i)$$

Now,  $OQ \perp TQ$

$$\text{In } \triangle OQT, OT^2 = OQ^2 + TQ^2 \quad [\text{By Pythagoras Theorem}]$$

$$\Rightarrow (x + 3)^2 = 5^2 + y^2$$

$$\Rightarrow x^2 + 9 + 6x = 25 + y^2$$

$$\Rightarrow x^2 + 9 + 6x = 25 + x^2 + 16 \quad [\text{By (i)}]$$

$$\Rightarrow 6x = 32 \Rightarrow x = \frac{32}{6} = \frac{16}{3}$$

$$\therefore y^2 = \left(\frac{16}{3}\right)^2 + 16 = \frac{256 + 144}{9} = \frac{400}{9}$$

$$\Rightarrow y = \frac{20}{3}$$

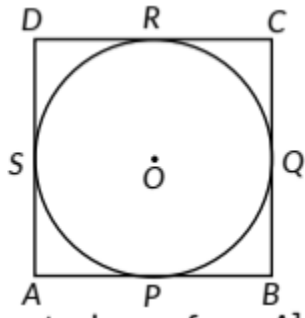
$$\text{Hence, } TP = TQ = \frac{20}{3} \text{ cm}$$

[ $\therefore$  Tangents drawn from an external point to a circle are equal in length]

65. Given: A parallelogram ABCD circumscribing a circle with centre O.

To prove : ABCD is a rhombus.

Proof: We know that the tangents drawn to a circle from an external point are equal in length.



$$\Rightarrow AP = AS \text{ [Tangents drawn from A] ... (i)}$$

$$\Rightarrow BP = BQ \text{ [Tangents drawn from B] ... (ii)}$$

$$\Rightarrow CR = CQ \text{ [Tangents drawn from C] ... (iii)}$$

$$\Rightarrow DR = DS \text{ [Tangents drawn from D] ... (iv)}$$

Adding (i), (ii), (iii) and (iv) we get

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow AB + CD = AD + BC$$

$$\Rightarrow 2AB = 2BC \text{ [Opposite sides of the given parallelogram are equal. } ABDC \text{ and } AD = BC]$$

$$\therefore AB = BC = DC = AD$$

Hence, ABCD is a rhombus.

$$66. PM = PN$$

(∵ Tangents drawn from an external point are equal)

$$PQ + QM = PR + RN$$

$$PQ + QS = PR + RS \text{ ... (i)}$$

$$\text{Now, Perimeter of } \triangle PQR = PQ + QR + PR$$

$$\Rightarrow PQ + QR + PR = PQ + QS + SR + PR = PQ + QS + PQ + QS$$

$$= 2(PQ + QS) = 2(PQ + QM) = 2PM$$

$$\Rightarrow PM = \frac{1}{2}(PQ + QR + PR)$$

Hence proved.

67. Construction: Join B and P

PQ and BQ are tangents to the circle centred at O from external point Q.

$$\Rightarrow PQ = BQ \text{ ... (i) [Length of tangents drawn from external point to the circle are equal]}$$

In  $\Delta PBQ$ ,  $\angle PBQ = \angle BPQ$  .....(ii)

AB is the diameter of the circle

$$\therefore \angle APB = 90^\circ \quad [\text{Angle in a semi circle is } 90^\circ]$$

$$\text{Now, } \angle APB + \angle BPC = 180^\circ \quad [\text{Linear pair}]$$

$$\Rightarrow \angle BPC = 180^\circ - 90^\circ = 90^\circ \quad \dots(\text{iii})$$

Now In  $\Delta BPC$

$$\angle BPC + \angle PBC + \angle PCB = 180^\circ$$

$$\Rightarrow \angle PBC + \angle PCB = 180^\circ - 90^\circ = 90^\circ \quad \dots(\text{iv})$$

$$\text{Now, } \angle BPQ + \angle CPQ = 90^\circ \quad [\text{Using (iii)}] \quad \dots(\text{v})$$

$$\angle PBC + \angle PCB = \angle BPQ + \angle CPQ$$

$$\Rightarrow \angle PCQ = \angle CPQ$$

$$[\because \angle BPQ = \angle PBQ, \angle PCB = \angle PCQ, \angle PBQ = \angle PBC]$$

In  $\Delta PQC$

$$\angle PCQ = \angle CPQ$$

$$PQ = QC$$

From (i) and (iv), we get  $BQ = QC$

Hence, PQ bisects BC.

68. Let common tangent at P meets the tangent AB at C.

Since, tangents drawn from an

external point to a circle are

equal.

$$\therefore AC = CP$$

$$\text{and } BC = CP$$

$$\Rightarrow \angle CAP = \angle CPA = x \text{ (say)} \quad \dots(\text{i})$$

$$\text{and } \angle CBP = \angle CPB = y \text{ (say)} \quad \dots(\text{ii})$$

$$\text{Now, } \angle ACP + \angle BCP = 180^\circ \quad [\text{Linear pair}] \quad \dots(*)$$

$$\text{In } \Delta ACP, \angle ACP + \angle CPA + \angle CAP = 180^\circ \quad \dots(\text{iii})$$

$$\text{and in } \Delta BCP, \angle BCP + \angle CPB + \angle CBP = 180^\circ \quad \dots(\text{iv})$$

Adding (iii) and (iv), we get

$$\angle ACP + x + x + \angle BCP + y + y = 360^\circ$$

[Using (i) & (ii)]

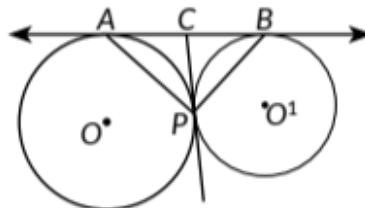
$$\Rightarrow \angle ACP + \angle BCP + 2x + 2y = 360^\circ$$

$$\Rightarrow 2(x + y) = 360^\circ - 180^\circ = 180^\circ$$

[Using (\*)]

$$\Rightarrow x + y = 90^\circ$$

$$\text{i.e., } \angle CPA + \angle CPB = 90^\circ \Rightarrow \angle APB = 90^\circ$$



69.

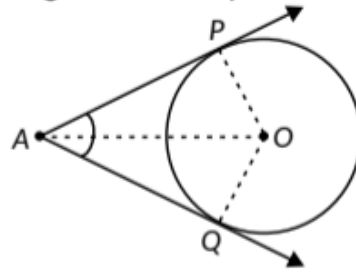
Given :  $AP$  and  $AQ$  are two tangents from a point  $A$  to a circle  $C(O, r)$

To Prove :  $AP = AQ$

Construction : Join  $OP$ ,  $OQ$  and  $OA$

Proof : In order to prove that  $AP = AQ$ , we shall first prove that  $\triangle OPA \cong \triangle OQA$

Since a tangent at any point of a circle is perpendicular to the radius through the point of contact.



$\therefore OP \perp AP$  and  $OQ \perp AQ$

$\angle OPA = \angle OQA = 90^\circ \dots(i)$

Now, in right triangle  $OPA$  and  $OQA$ , we have

$OP = OQ$  [Radii of circle]

$\angle OPA = \angle OQA$  [Each  $90^\circ$ ]

and  $OA = OA$  [Common]

So, by RHS-criterion of congruence, we get

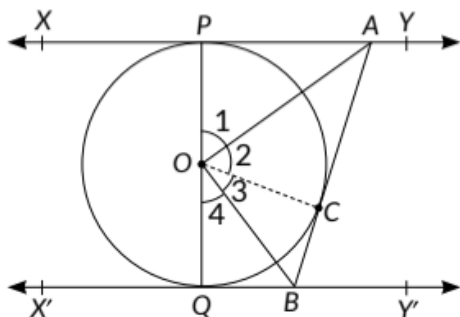
$\triangle OPA \cong \triangle OQA \Rightarrow AP = AQ$  [By CPCT]

Hence, lengths of two tangents drawn from an external point are equal.

70. Given: Two parallel tangents  $XY$  and  $X'Y'$  to a circle with centre  $O$  and another tangent  $AB$  with point of contact  $C$  intersecting  $XY$  at  $A$  and  $X'Y'$  at  $B$ .







To prove:  $\angle AOB = 90^\circ$

Proof: In  $\triangle APO$  and  $\triangle ACO$ ,

$AP = AC$  (Tangents drawn from an external point are equal in length)

$OP = OC$  (Radii of same circle)

$OA = OA$  (Common)

$\therefore \triangle APO \cong \triangle ACO$  (SSS congruence criterion)

$\therefore \angle 1 = \angle 2$  (By CPCT) ... (i)

Similarly,  $\triangle OCB \cong \triangle OQB$

$\therefore \angle 3 = \angle 4$  (By CPCT) ... (ii)

Now,  $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$  (Angles on a straight line)

$\Rightarrow 2\angle 2 + 2\angle 3 = 180^\circ$  (Using (i) and (ii))

$\Rightarrow \angle 2 + \angle 3 = \frac{180^\circ}{2} = 90^\circ \Rightarrow \angle AOB = 90^\circ$

71. We know, tangent to a circle is perpendicular to its radius at the point of contact.

So,  $OP \perp PT$  and  $OQ \perp QT$

In  $\triangle OPT$ ,  $(OP)^2 + (PT)^2 = OT^2 \Rightarrow PT^2 = (OT)^2 - (OP)^2$

$\Rightarrow (PT)^2 = 169 - 25 = 144 \Rightarrow PT = 12 \text{ cm}$

$\Rightarrow PT = QT = 12 \text{ cm}$

( $\because$  Tangents drawn from an external point are equal)

Let  $PA = x \text{ cm} \Rightarrow AT = (12 - x) \text{ cm}$

Hence, in right angled  $\triangle AET$

$(AE)^2 + (ET)^2 = (AT)^2$  ( $\because OE \perp AB$ )

$\Rightarrow (x)^2 + (8)^2 = (12 - x)^2$  ( $\because PA = AE$  and  $ET = OT - OE$ )

$\Rightarrow x^2 + 64 = 144 + x^2 - 24x$

$\Rightarrow x = \frac{80}{24} = 3.33$

In  $\triangle AET$  and  $\triangle BET$

$\angle ETA = \angle ETB$

$ET = ET$  (Common)

$\angle AET = \angle BET$  ( $90^\circ$  each, as  $OE \perp AB$ )

$\Rightarrow \triangle AET \cong \triangle BET$  (By ASA congruence)

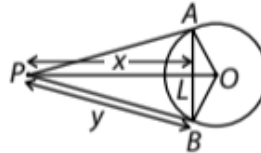
$\Rightarrow AE = EB$  (By CPCT)



Now,  $AB = AE + EB$   
 $= AB = AE + AE$   
 $= AB = 2AE = 2 \times 3.33 = 6.66 \text{ cm}$

72.

We have,  $AB = 16 \text{ cm}$   
 $\therefore AL = BL = 8 \text{ cm}$   
 In  $\triangle OLB$ , we have  
 $OB^2 = OL^2 + LB^2$   
 $\Rightarrow 10^2 = OL^2 + 8^2 \Rightarrow OL^2 = 100 - 64 = 36$   
 $\Rightarrow OL = 6 \text{ cm}$



Let  $PL = x \text{ cm}$  and  $PB = y \text{ cm}$   
 Then,  $OP = (x + 6) \text{ cm}$   
 In  $\triangle PLB$ ,  $PB^2 = PL^2 + BL^2 \Rightarrow y^2 = x^2 + 64$   
 Now,  $OB \perp PB$ .

In  $\triangle OBP$ ,  $OP^2 = OB^2 + PB^2$   
 $\Rightarrow (x + 6)^2 = 100 + y^2$   
 $\Rightarrow x^2 + 36 + 12x = 100 + x^2 + 64 \quad [ \because y^2 = x^2 + 64 ]$

$$\Rightarrow 12x = 128 \Rightarrow x = \frac{32}{3}$$

$$\therefore y^2 = \left(\frac{32}{3}\right)^2 + 64 = \frac{1600}{9} \Rightarrow y = \frac{40}{3}$$

Hence,  $PA = PB = \frac{40}{3} \text{ cm}$

73. We know that, tangents drawn from an external point to a circle are equal in length.

So,  $PQ = PR$

$\therefore \angle PRQ = \angle PQR$

In  $\triangle PQR$ ,

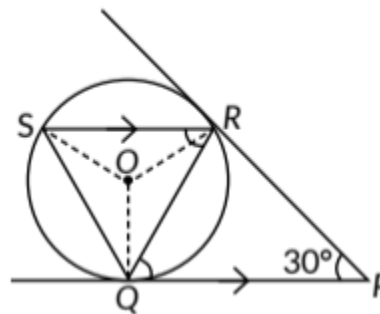
$\angle PQR + \angle PRQ + \angle QPR = 180^\circ$

$\Rightarrow \angle PQR + \angle PQR + 30^\circ = 180^\circ$

$\Rightarrow 2\angle PQR = 150^\circ \Rightarrow \angle PQR = 75^\circ$

Since  $SR \parallel QP$  and  $QR$  is a transversal,

$\therefore \angle SRQ = \angle PQR = 75^\circ$



Now, join  $OR$ ,  $OS$  and  $OQ$

We know that, angle subtended by an arc at the centre is double the angle

subtended by same arc at any point on the remaining part of the circle.

$$\text{So, } \angle SOQ = 2\angle SRQ \Rightarrow \angle SOQ = 2 \times 75^\circ = 150^\circ$$

Now, in  $\triangle OSQ$ ,  $OS = OQ$

$$\Rightarrow \angle OQS = \angle OSQ$$

Now, by angle sum property,

$$2\angle OQS = 180^\circ - 150^\circ \Rightarrow \angle OQS = 15^\circ$$

We know that, tangent is perpendicular to radius at the point of contact.

$$\text{So, } \angle OQP = 90^\circ$$

$$\Rightarrow \angle OQR + \angle PQR = 90^\circ$$

$$\Rightarrow \angle OQR = 90^\circ - 75^\circ = 15^\circ$$

Now,  $\angle RQS = \angle OQR + \angle OQS$

$$\Rightarrow \angle RQS = 15^\circ + 15^\circ = 30^\circ$$

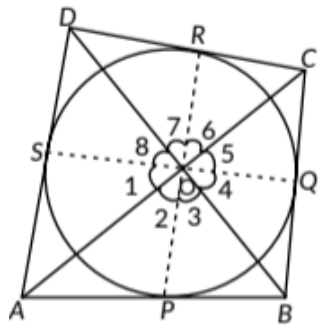
74. Given: ABCD is a quadrilateral circumscribing a circle whose sides AB, BC, CD and DA touches the circle at P, Q, R and S respectively.

To prove :  $\angle AOB + \angle COD = 180^\circ$

and  $\angle BOC + \angle AOD = 180^\circ$

Construction Join OP, OQ, OR and OS.

Proof : Since we know that tangents drawn from an external point to a circle subtend equal angles at the centre.



$$\begin{aligned} \therefore \angle 1 &= \angle 2, \\ \angle 3 &= \angle 4, \\ \angle 5 &= \angle 6 \text{ and} \\ \angle 7 &= \angle 8 \end{aligned}$$

$$\text{Now, } \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

[Sum of all angles around a point is  $360^\circ$ ]

$$\Rightarrow 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360^\circ \text{ and}$$

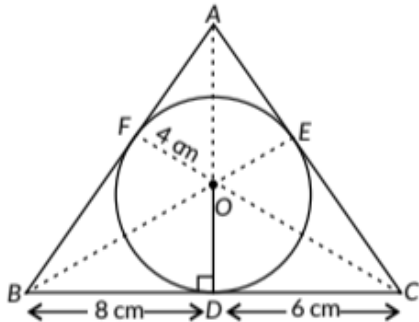
$$2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360^\circ$$

$$\Rightarrow (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^\circ \text{ and}$$

$$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$$

$$\Rightarrow \angle AOB + \angle COD = 180^\circ \text{ and } \angle AOD + \angle BOC = 180^\circ$$

75. Given,  $\triangle ABC$  circumscribing a circle of radius 4 cm with centre O.  
 Join OA. Draw BE and CF such that these pass through the centre O.  
 Now, radius  $OD = OE = OF = 4$  cm



Since, tangents drawn from an external point are equal

Also  $BD = BF = 8$  cm,  $DC = EC = 6$  cm

$AF = AE = x$  cm (say)

Semi-perimeter of  $\triangle ABC$ ,

$$s = \frac{(x+8) + (14) + (x+6)}{2} = (x+14) \text{ cm}$$

Area of  $\triangle ABC$

$$\begin{aligned} &= \sqrt{(x+14)(x+14-x-8)(x+14-14)(x+14-x-6)} \\ &= \sqrt{(x+14)(6)(x) \times 8} = \sqrt{48x(x+14)} \text{ cm}^2 \end{aligned} \quad \dots(i)$$

Also,

Area of  $\triangle ABC = \text{Area of } \triangle OAB + \text{Area of } \triangle OBC + \text{Area of } \triangle OAC$

$$\begin{aligned} &= \frac{1}{2} \times (x+8) \times (4) + \frac{1}{2} \times (14) \times (4) + \frac{1}{2} \times (x+6) \times (4) \\ &= (4x + 56) \text{ cm}^2 \end{aligned} \quad \dots(ii)$$

From (i) and (ii), we have

$$\begin{aligned} \sqrt{48x(x+14)} &= 4x + 56 \\ \Rightarrow 48x(x+14) &= (4x+56)^2 && \text{(Squaring on both sides)} \\ \Rightarrow 48x(x+14) &= 16(x+14)^2 \\ \Rightarrow 3x(x+14) &= (x+14)^2 \Rightarrow 3x = x+14 \\ \Rightarrow 2x &= 14 \Rightarrow x = 7 \quad \therefore AB = x+8 = 7+8 = 15 \text{ cm} \\ \text{and } AC &= x+6 = 7+6 = 13 \text{ cm} \end{aligned}$$

## CBSE Sample Questions

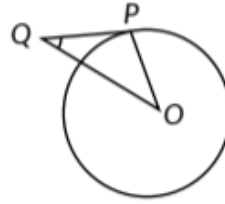
1.

In  $\triangle OPQ$   
 $\angle P + \angle Q + \angle O = 180^\circ$

$$\Rightarrow 2\angle Q + 90^\circ = 180^\circ \quad (1/2)$$

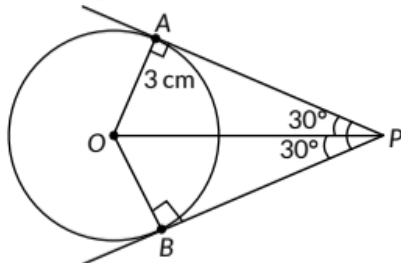
[ $\because \triangle OPQ$  is an isosceles triangle]

$$\Rightarrow 2\angle Q = 90^\circ \Rightarrow \angle Q = 45^\circ \quad (1/2)$$



2.

(d): From the figure,  $\angle PAO = 90^\circ$



In  $\triangle PAO$ ,  $\tan 30^\circ = \frac{AO}{AP} \quad (1/2)$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AP} \Rightarrow AP = 3\sqrt{3} \text{ cm} \quad (1/2)$$

3. Here  $PQ = PT$

[ $\because$  Length of tangents drawn from an external point to the circle are equal]

$$= PL + LQ = PM + MT$$

$$= PL + LN = PM + MN \dots (i)$$

Now, perimeter ( $\triangle PLM$ )  $PL + LM + PM$  (1/2)

$$= PL + LN + MN + PM \quad [\because LM = LN + NM]$$

$$= 2(PL + LN) \quad [\text{Using (i)}]$$

$$= 2(PL + LQ) = 2 \times PQ$$

$$= 2 \times 28 = 56 \text{ cm} \quad [\because PQ = 28 \text{ cm (Given)}] \quad (1/2)$$

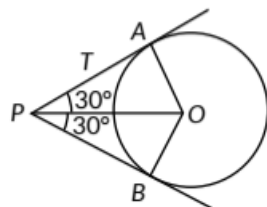
4.

In  $\triangle PAO$   
 $\tan 30^\circ = \frac{AO}{PA} \quad (1/2)$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{PA}$$

[ $\because$  Radius = 3 cm (Given)]

$$\Rightarrow PA = 3\sqrt{3} \text{ cm} = PB$$



(1/2)

5. We have,  $\angle ZPAO = \angle ZPBO = 90^\circ$  (1)

(angle between radius and tangent)

Since, APBO is quadrilateral and  $\angle ZAP + \angle ZPB + \angle ZOB + \angle ZOA = 360^\circ$

$$= 90^\circ + 75^\circ + 90^\circ + \angle ZOB = 360^\circ \Rightarrow \angle ZOB = 105^\circ \quad (1/2)$$

$\angle AQB = \frac{1}{2} \times 105^\circ = 52.5^\circ$  (Angle at the remaining part of the circle is half the angle subtended by the arc at the centre) (1/2)

6.

Let  $\angle APO = \theta$

$$\Rightarrow \sin \theta = \frac{OA}{OP} = \frac{1}{2} = \sin 30^\circ \Rightarrow \theta = 30^\circ \quad (1/2)$$

$$\angle APB = 60^\circ \quad (1/2)$$

Also,  $\angle ZPAB = \angle ZPBA = 60^\circ$  ( $\because PA = PB$ ) (1/2)

$\Rightarrow \triangle APB$  is an equilateral triangle. (1/2)

7. Clearly,  $\angle ZAPO = \angle ZASO = 90^\circ$  (1/2)

$$\angle SOP = 90^\circ$$

Also,  $AP = AS$

[Length of tangents from the external point A]

So, OSAP is a square

$$\Rightarrow AP = AS = 10 \text{ cm [ OS 10 cm (Given) ]} \quad (1/2)$$

Again,  $CQ = 27 \text{ cm}$

[Length of tangents from the external point C]

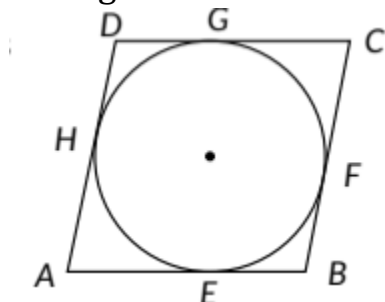
$$\Rightarrow BQ = BC - CQ = 38 - 27 = 11 \text{ cm}$$

$$\Rightarrow BP = BQ = 11 \text{ cm} \quad (1/2)$$

$$\therefore x = AB = AP + BP = 10 + 11 = 21 \text{ cm} \quad (1/2)$$

8. Let ABCD be the rhombus circumscribing the circle with centre O, such that AB, BC, CD and DA touch the circle at points E, F, G and H respectively.

We know that the tangents drawn to a circle from an exterior point are equal in length.



$$\therefore AE = AH \quad \dots(i)$$

$$BE = BF \quad \dots(ii)$$

$$CG = CF \quad \dots(iii)$$

$$DG = DH \quad \dots(iv)$$

Adding (i), (ii), (iii) and (iv) we get

$$AE + BE + CG + DG = AH + BF + CF + DH$$

$$\therefore AB + CD = AD + BC \quad \dots(v)$$

Since  $AB = DC$  and  $AD = BC$

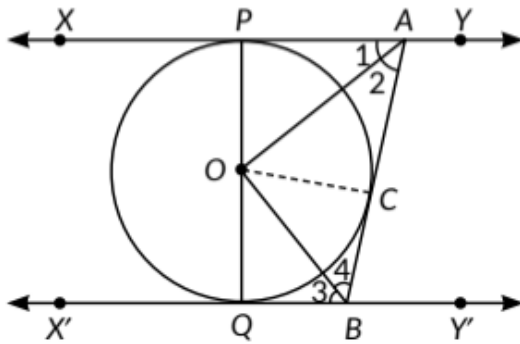
$$\text{Putting in (v) we get, } 2AB = 2AD \quad (1)$$

$$\text{or } AB = AD$$

$$\therefore AB = BC = DC = AD$$

Since a parallelogram with equal adjacent sides is a rhombus, so  $ABCD$  is a rhombus. (1)

9.



Join OC

In  $\triangle OPA$  and  $\triangle OCA$

$OP = OC$  (radii of same circle)

$PA = CA$  (length of two tangents from an external point)

$AO = AO$  (Common)

Therefore,  $\triangle OPA \cong \triangle OCA$  (By SSS congruency criterion)

Hence,  $\angle 1 = \angle 2$  (CPCT) (1)

Similarly  $\angle 3 = \angle 4$

$\angle PAB + \angle QBA = 180^\circ$  (co-interior angles are supplementary as  $XY \parallel X'Y'$ )

$$\Rightarrow 2\angle 2 + 2\angle 4 = 180^\circ$$

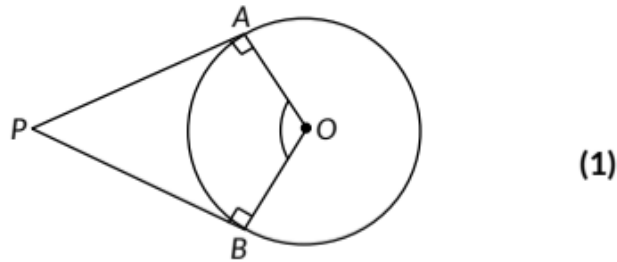
$$\Rightarrow \angle 2 + \angle 4 = 90^\circ \quad \dots(i) \quad (1)$$

In  $\triangle AOB$ ,

$$\angle 2 + \angle 4 + \angle AOB = 180^\circ \text{ (Angle sum property)} \quad (1)$$

Using (i), we get  $\angle AOB = 90^\circ$

10. Let PA and PB be the two tangents drawn to a circle with centre O.



(1)

$\Rightarrow \angle OAP = \angle OBP = 90^\circ$  (Tangents to a circle)

To prove:  $\angle APB + \angle AOB = 180^\circ$

Now,  $\angle OAP + \angle OBP + \angle APB + \angle AOB = 360^\circ$   $(1\frac{1}{2})$

$\Rightarrow 90^\circ + 90^\circ + \angle APB + \angle AOB = 360^\circ$

$\Rightarrow \angle APB + \angle AOB = 180^\circ$   $(1\frac{1}{2})$

Hence proved.

11.

Let  $\angle PTQ = \theta$

Here,  $\triangle TPQ$  is an isosceles triangle. ( $\because TP = TQ$ )

$\Rightarrow \angle TPQ = \angle TQP = \frac{1}{2}(180^\circ - \theta) = 90^\circ - \frac{\theta}{2}$   $(1\frac{1}{2})$

Also,  $\angle OPT = 90^\circ$  ( $\because$  Tangent to a circle is perpendicular to the radius through the point of contact)

$\Rightarrow \angle OPQ = \angle OPT - \angle TPQ = 90^\circ - \left(90^\circ - \frac{\theta}{2}\right) = \frac{\theta}{2}$   $(1\frac{1}{2})$

$\Rightarrow \angle OPQ = \frac{1}{2} \angle PTQ$

$\Rightarrow \angle PTQ = 2 \angle OPQ$  (1)

Hence proved.